Implementation of Improved Design of Costas Carrier Recovery Loop for Coherent Demodulation

Hina Shahid, Tahir Izhar, Sadia Murawwat

Abstract – In autonomous radio, receivers contain carrier synchronization structure that is capable of tracking carrier phase and frequency information independently. In this paper, that configuration is presented using improved design of Costas loop consists of Hilbert filter and negative feedback path for carrier regeneration. Costas loop is a closed loop scheme that successfully track fully suppressed carrier signals. The objective of this paper is to evaluate the performance of improved design of Costas loop. A mathematical modeling was developed for Costas loop presented in this paper and evaluated by using MATLAB and then implemented using SIMULINK for AM (Amplitude modulation) and BPSK (Binary Phase Shift keying) signals. The loop remove phase offsets completely and frequency offsets up to 3080 Hz.

Index Terms— Costas loop; Demodulation; Hilbert filter; AM; BPSK; MATAB; SIMULINK

I. INTRODUCTION

In digital communication, synchronization technique used in coherent receivers, shown in Fig.1, for information recovery.[1] Synchronization-based receivers have advantages over non-coherent receivers in terms of noise performance and bandwidth efficiency. It requires three levels i.e. carrier synchronization, symbol main synchronization and frame synchronization.[2] In this paper, our focus is on carrier synchronization of receiver as its failure causes catastrophic effects on the performance of communication system and hence other two levels of synchronization cannot be achieved. [3-5]

Phase and frequency of received modulated signal (high frequency signal) at the receiver end must match with its feedback signal for coherent detection in carrier synchronization. The standard approach to this technique is a Phase Locked Loop (PLL), shown in Fig. 2.



Fig. 1. Coherent Receiver [3]

It is a feedback loop, can be utilized for carrier synchronization, carrier recovery and frequency synthesizing, as a synchronizer it tracks only residual carrier signals and positive energy in carrier wasted as it does not transmitted.[6] Narrow bandwidth and long acquisition time are problems in its good performance. To overcome these drawbacks there was entail to improve carrier synchronization loop. [4, 7]



In 1956, John Costas invented novel synchronization loop that not only recover carrier information (phase and frequency) but it can also detect data of incoming suppressed carrier signals at receiver, known as Costas loop. [8]

Traditional design of Costas loop [9], shown in Fig. 3, involves two parallel tracking loops that are 90 degrees phase shifted from each other. Upper loop is called inphase loop (I-arm) and lower loop is called quadrature loop (Q-arm). Low pass filters in each arm must be wide enough to pass the carrier modulation without any distortion. In this design of Costas loop perfect matching of two low pass filters used in I-arm and Q-arm for filtration is not possible. [10, 11]

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Fig. 3. Costas Loop [9]

There are three types of Costas loop with respects to integrator in its feedback path

- 1st order Costas loop
- 2nd order Costas loop
- 3rd order Costas loop

In [12], a modified design of 2nd order Costas loop, shown in Fig.4, is presented that replaces two arm filters with Hilbert filter and carrier is regenerated using feedback path. 2nd order Costas loop has an ability to track not only phase (1st order) of incoming modulated signal as well as constant frequency offset (2nd order). [13]

In this paper, implementation of 2nd order Costas loop is presented for high data rate signals.



Fig. 4. Improved Design of Costas Loop

II. WORKING PRINCIPLE OF IMPROVED DESIGN

The received signal at input side of Costas loop , shown in Fig 4, passes from Hilbert filter that transformed this signal into its analytical form and this analytical signal is multiplied by a complex exponential exp($-j\omega_c t$) that is form by using an estimate of the carrier signal's frequency and phase.[14] Hilbert filter, analytical signal and complex envelope are explained here one by one.

A. Hilbert Filter

Hilbert filter imparts a $-\pi/2$ (-90°) phase shift in the input signal without modifying the magnitude of the input signal. Mathematically it expressed in (1).

$$H(\cos \alpha) = \sin \alpha \tag{1}$$

The Hilbert filter has been introduced is the key to generate analytic signals in number of Digital Signal Processing (DSP) applications.

B. Analytical Signal

Analytical signal, shown in Fig. 5, is defined as the original signal (incoming signal) plus j times the hilbert transform of original signal. In the frequency domain, the construction of an analytic signal has the effect of eliminating the negative frequency components of original signal and doubling the positive frequency components.



C. Complex Envelop

The complex envelope is formed by multiplying the analytical signal by the complex exponential exp ($-j\omega_c t + \theta_2$), where ω_c and θ_2 are the loop's estimate of the carrier frequency and carrier phase respectively. Complex signal in the frequency domain is shown in Fig. 6.



If the frequency and phase of carrier estimated accurately, the real components of the complex envelope represent the message signal. [15]

III. MATHEMATICAL MODELING

To evaluate the performance of costas loop, shown in Fig. 4, mathematical modeling is developed that starts with incoming signal x(nTs) at the input side of Costas loop.

$$x(nTs) = A_c m(nTs) \cos(\omega_c nTs + \theta_1)$$
(2)

Where, ω_c is the carrier frequency and θ_1 is a phase angle. This incoming signal (2) passes from Hilbert filter of Costas loop that generate -90° shifted signal expressed in (3);

$$H[x(nTs)] = h(nTs) = jA_c m(nTs) sin(\omega_c nTs + \theta_1)$$
(3)

Analytic signal of (3) represented by $P^+(t)$ can be constructed as

$$P^{+}(nTs) = x(nTs) + jh(nTs)$$
(4)

$$P^{+}(nTs) = A_{c} m(nTs) [\cos(\omega_{c}nTs + \theta_{1}) + j\sin(\omega_{c}nTs + \theta_{1})]$$
(5)

$$P^{+}(nTs) = A_{c} m(nTs)e^{j(\omega cnT+\theta 1)}$$
(6)

Complex envelope is formed by multiplying the analytical signal by the complex exponential exp $(-j\omega_c nTs + \theta_2)$ that is the estimate signal generated from feedback path;

$$C(nTs) = A_c m(nTs) e^{j(\omega cnTs + \theta 1)} e^{-j(\omega cnTs + \theta 2)}$$
(7)

$$C(nTs) = A_c m(nTs) e^{j[\theta 1 - \theta 2]}$$
(8)

$$C(nTs) = A_c m(nTs)[\cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2)]$$
(9)

Where,

$$Cr(nTs) = A_c m(nTs) \cos (\theta_1 - \theta_2)$$
(10)

Real part (I-arm signal)

$$Ci(nTs) = A_c m(nTs) sin (\theta_1 - \theta_2)$$
(11)

Imaginary part (Q-arm signal)

The real and imaginary components of signal pass from Phase detector and generate error signal;

$$e(nTs)=Cr(nTs) . Ci(nTs)$$
(12)

$$e(nTs) = A_c^2 m^2 (nTs) [\cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2)]$$
(13)

$$e(nTs)=1/2[A_c^2 m^2(nTs) \sin^2(\Delta \theta)]$$
(14)

Error signal controls regeneration of carrier signal through loop filter. After several iterations phase error $\Delta \theta$ is reduced and approaches to zero and loop is locked. When the loop is in locking condition, the small angle approximation, sin x=x, can be used to accurately approximate e(nTs). [16]

$$e(nTs) = A_c^2 m^2 (nTs) [\theta_1 - \theta_2]$$
(15)

Locked condition of costas loop

Cr(nTs) = m(nTs) Original message signal (16)

 $Ci(nTs) = 0 \tag{17}$

IV. LOOP FILTER

Loop filter is key part of Costas loop. Loop filter not only filter the noise in error measurements of phase detector but also track error. Loop filter can be continuous time or discrete time. In this paper, discrete time loop filter, shown in Fig.7, is implemented to track the phase error and frequency error of incoming signal. Loop filter passes low frequency components of the error signal while block the higher frequency components of the error signal. [12, 14]



Fig. 7. Loop Filter

Transfer function of loop filter is found by applying simple control theory and it represented in (18)

$$H(z) = \frac{k_1 \beta + k_1 \alpha \left(1 - \frac{\alpha}{\alpha + \beta} z^{-1}\right)}{1 - \left\{2 - k_1 \left(\alpha + \beta\right)\right\} z^{-1} + \left(1 - k_1 \alpha\right) z^{-2}}$$
(18)

The error signal in (14) pass from loop filter and output of loop filter is controlled by its coefficients α and β , there values can be calculated as

$$\alpha k_1 = 2\xi \left(\frac{2B_L T_S}{\xi + \frac{1}{4\xi}} \right)$$
(19)

$$\beta k_1 = \left(\frac{2B_L T_S}{\xi + \frac{1}{4\xi}}\right) \tag{20}$$

Where, B_L is loop bandwidth and define as "range of frequencies that the loop filter passes". ξ is the loop damping factor and its optimal value ξ =0.707 used during implementation of loop.

V. VOLTAGE CONTROL OSCILLATOR

In this paper, VCO is completely implemented in discrete time domain by integrating the output of loop filter and complex envelop signal is generated.

VI. IMPLEMENTATION OF IMPROVED DESIGN

The parameters used to implement Costas loop are summarized in Table I. To evaluate the performance of design of Costas carrier recovery loop presented in this paper, MATLAB software is used. Mathematical modeling presented in III is tested in MATALB for AM and BPSK signals, results shown in Fig. 8 and Fig. 9. Costas loop presented in this paper can handled phase ambiguity very well, this can be seen in Fig. 9 when Qchannel signal (imaginary signal) tried to unlock from loop at 180° phase change of input signal but Costas loop will not anti lock and still detect data.

TABLE I. LOOP PARAMETERS

Symbol	Parameter	Value
m(nTs)	Message Signal	200 Hz
c(nTs)	Carrier Signal	2000 Hz
\mathbf{B}_{L}	Loop bandwidth	1000 Hz
Ts	Sampling Time	16000 s
ξ	Damping Factor	0.707
α	Loop Coefficient	0.33
β	Loop Coefficient	0.02778



Fig. 8. Demodulation of Input AM signal



Fig. 9. Demodulation of Input BPSK signal

Simulations are used to imitate the behavior of actual hardware. Simulink based model of Costas loop shown in Fig. 10. Externally generated modulated signal, shown in Fig. 11, passes from Hilbert filter that imparts 90°, shown in Fig. 12.



Fig. 10. SIMULINK Model of Design



Fig. 11. Incoming Modulated Signal



Fig. 12. Hilbert Filter Output

After hilbert filter, complex multiplier used to generate I and Q channel signals by using feedback estimated carrier signal. Implementation of complex multiplier shown in Fig. 13



Fig. 13. Complex Multiplier

I and Q channel signals passes from phase detector that generate error signal. Memory-less phase detector used in Costas loop that consists of multiplier whose output consist of multiplication of input signals. This type of detector use in high frequency applications as they can provide the product of high frequencies applied to its input.[17] Phase detector output shown in Fig. 14.



Fig. 14. Phase Detector Signal

Costas loop is a time-varying system because of the sin and $m^2(nTs)$ terms in q(nTs). Therefore, it cannot be characterized by a transfer function. However, when loop is in lock, it can be accurately approximated by timeinvariant system by using the small angle approximation presented in (15) then loop filter act as low pass filter on q(nT), shown in Fig. 15.



Fig. 15. Loop Filter Signal

Estimated feedback carrier signal generated after loop filter passes from VCO that was implemented completely in digital environment, its output signal shown in Fig. 16 and Fig. 17 in time and frequency domain respectively.







Fig. 17. Regenerated Carrier Signal

According to (16) and (17), I and Q channel signals shown in Fig. 18.



Fig. 18. I and Q channel Signals

Costas loop performance further evaluated with different values of carrier to find its pull-in range. Fig. 19 and Fig. 21 shows loop performance for frequency offset of 100 Hz and 1500 Hz respectively. The design of Costas loop presented in this paper working optimally for frequency offsets of up to 3080Hz in 0.008 second.



Fig. 19. I and Q channel Signals for 100Hz Frequency offset



VII. CONCLUSION

Time by time innovation in technologies and market place dictates that the architecture of communication systems should be modified in design and cost perspective. This paper evaluates the implementation of carrier synchronization using optimized design of Costas loop for AM/BPSK signals in MATLAB/SIMULINK. During implementation of Costas loop, the main focus was on the implementation of Hilbert filter, phase detector and loop filter. This loop is working for loop bandwidth of 1000Hz and track carrier in 0.008s for the optimum value of damping factor 0.707. Costas loop presented in this paper not only remove phase offsets perfectly but also tracked constant frequency offset up to 3080 Hz. The simulation results show that this whole architecture results in robust and accurate carrier recovery than the traditional design of Costas loop.

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