New Techniques to Suppress the Sidelobes in OFDM System to Design a Successful Overlay System

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***Abstract*—In orthogonal frequency division multiplexing (OFDM) systems the reduction of out-of-band radiations is an essential topic. To design a successful OFDM based overlay systems, it is important to reduce the sidelobes in OFDM systems. In this paper we consider three approaches: Subcarrier weighting (SW) which is based on the multiplication of the used subcarriers with real-valued weighting factors, second approach we consider is multiple choice sequence (MCS) which is based on producing set of sequences and selecting the one sequence which has lowest power in sidelobes and third approach we consider is the conventional windowing of OFDM signal in time domain. We combine MCS with windowing technique and SW with windowing technique. Simulation results show that by combining MCS with windowing technique and SW with windowing technique, the out-of-band radiations in OFDM system can be significantly suppressed.**

***Index Terms*— Orthogonal frequency division multiplexing, out-of-band radiations, sidelobe suppression**

I. INTRODUCTION

In Europe orthogonal frequency division multiplexing (OFDM) has been successfully used by standards such as the digital audio broadcasting and the digital video broadcasting for modulation. The main advantage of OFDM is to avoid the intersymbol interference and intercarrier interference within an OFDM symbol with a small loss of transmission power by using the concept of a cyclic prefix. In OFDM system individual subcarriers can be switched on or off, which makes OFDM system is very attractive to implement in so called spectrum sharing systems [1]. The main drawback in designing OFDM based overlay system is the out-of-band radiations generated by OFDM transmission signal. As in [3], Figure 1 shows the concept of co-existence between OFDM based overlay system and existing legacy system in frequency band assigned to the existing legacy systems. A crucial task in designing such an overlay system is the avoidance of interference towards the existing legacy systems. Therefore, the sidelobe suppression has been an essential topic.

An overview of the existing techniques for sidelobe suppression can be found in [2]. The first method in [2] is based on the insertion of guard bands and the second method is based on windowing of the transmission signal in frequency domain. Method in [3] is known as subcarrier

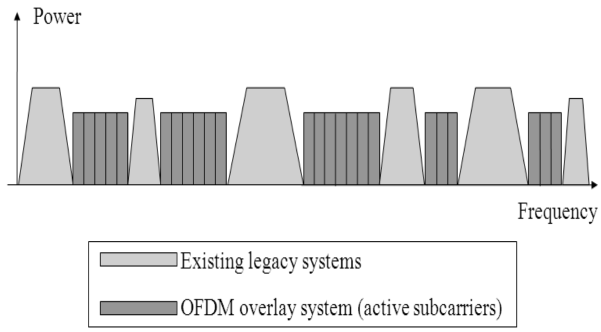

Fig.1. OFDM system within frequency band assigned to existing systems.

weighting (SW), which is based on the multiplication of the used subcarriers with real-valued weighting factors which are chosen such that the sidelobes are suppressed. Another method for sidelobe suppression which insert a few subcarriers, known as cancellation carriers [4] at both sides of the spectrum which are weighted such that their sidelobes cancel the sidelobes of the transmit signal. Multiple choice sequence (MCS) [5] which is based on producing set of sequences and selecting the one sequence which has lowest power in sidelobes.

In [6] the combination of windowing with cancellation carriers is proposed and analyzed. In this paper, we combine SW with windowing technique and also we combine MCS with windowing technique. Results show that by combining SW with windowing technique and MCS with windowing technique, the out-of band radiations in OFDM system can be significantly reduced which enables to design successful OFDM based overlay system.

The paper is organized as follows. In Section 2, the system model is described. In Section 3, the sidelobe suppression techniques are explained. Section 4 contains the numerical results. Finally, Section 5 is the conclusion.

II. SYSTEM MODEL

The block diagram of the OFDM transmitter which includes combination of MCS with windowing and combination of SW with windowing are illustrated in Figure 2 and Figure 3, respectively. We consider an OFDM system with total number of N subcarriers. The input bits are symbol-mapped by applying the modulation technique of phase-shift keying (PSK) or quadrature amplitude modulation (QAM) and N data symbols $d_n, n = 1, 2, ..., N$, are generated and then these symbols are serial-to-parallel (S/P) converted which results into an vector $\mathbf{d} = (d_1, d_2, ..., d_N)^T$ where $(.)^T$ denotes transposition. As

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illustrated in Figure 2, the vector $\mathbf{d}$ is fed into the MCS sidelobe suppression unit, which outputs the sequence denoted by $\mathbf{r}=(r_1,r_2,...,r_N)^T$. The output of MCS sidelobe suppression unit $\mathbf{r}$ is modulated onto N subcarriers using the inverse discrete Fourier transform (IDFT). After that, parallel-to-serial (P/S) conversion is performed. Then, cyclic prefix is added. Next, signal is digital-to-analog (D/A) converted. After that, D/A converted time domain signal is multiplied with a windowing function $w^{RC}(t)$. As depicted in Figure 3, the vector $\mathbf{d}$ is fed into the SW sidelobe suppression unit, which outputs the sequence denoted by $\mathbf{q}=(q_1,q_2,...,q_N)^T$. Resulting sequence $\mathbf{q}$ is modulated onto N subcarriers using the IDFT. After that, P/S conversion is performed, cyclic prefix is added with P/S converted signal, then signal is D/A converted. Next, D/A converted time domain signal is multiplied with a windowing function $w^{RC}(t)$.

III. Sidelobe Suppression Techniques

A. Sidelobe Suppression by MCS

The MCS [5] technique is based on producing set of mapped sequences from the original transmission sequence and choosing the one sequence from the MCS set for transmission which has lowest power in the sidelobes. The MCS system has two parts, first part generates MCS sets and the second part chooses the sequence which has lowest power in sidelobes.

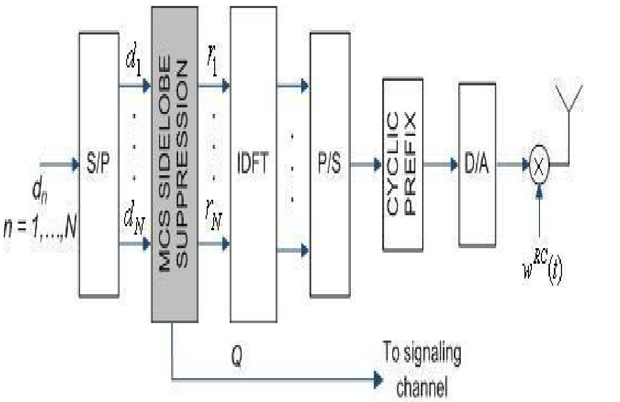

Fig.2. Block diagram of the OFDM transmitter with MCS and windowing.

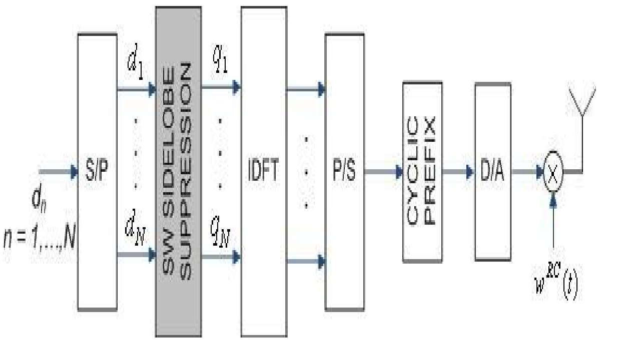

Fig.3. Block diagram of the OFDM transmitter with SW and windowing.

A single subcarrier in frequency domain is represented as

$$s_n(f)=d_n.\mathbf{si}(\pi(f-f_n)T_0),\ n=1,2,...,N \quad (1)$$

where f denotes the frequency, f_n is the carrier frequency of the n^{th} subcarrier, and T_0 is the OFDM symbol duration including guard time T_G, i.e. $T_0=T_s+T_G$, where T_s is the OFDM symbol duration without guard time. The spectrum of each subcarrier is equal to a si-function which is defined as $\mathbf{si}(x)=\sin(x)/x$, where x is the normalized frequency.

Using MCS a set of P > 1 sequences, $\mathbf{d}^{(p)}=(d_1^{(p)},d_2^{(p)},...,d_N^{(p)})^T$, $p=1,2,...,P$, are produced from the original data sequence $\mathbf{d}$. The average sidelobe power denoted with $\mathbf{A}^{(p)}$, $p=1,2,...,P$ is calculated for each MCS generated sequence $\mathbf{d}^{(p)}$. To determine the average sidelobe power, a certain frequency range called optimization range spanning several OFDM sidelobes are considered using discrete frequency samples. The optimization range is illustrated in Figure 4, the optimization range is divided in two approximately equal parts. $\mathbf{A}^{(p)}$ is given by

$$\mathbf{A}^{(p)}=1/K\sum_{k=1}^{K}\left|\sum_{n=1}^{N}d_n^{(p)}\frac{\sin(\pi(y_k-x_n))}{\pi(y_k-x_n)}\right|^2,\ p=1,2,...,P \quad n=1,2,...,N \quad k=1,2,...,K \quad (2)$$

In Equation (2), $x_n, n=1,2,...,N$ denotes the normalized subcarrier frequencies and K samples at the normalized frequencies $y_k, k=1,2,...,K$ are considered, which are in the frequency range where the optimization of the sidelobes is performed.

As explained in [5], the index Q of the selected sequence from the MCS set which has lowest power in sidelobes is given by

$$Q=\arg\min_{p}\mathbf{A}^{(p)},\ p=1,2,...,P \quad (3)$$

So, the sequence $\mathbf{d}^{(Q)}=(d_1^{(Q)},d_2^{(Q)},...,d_N^{(Q)})$ is the one selected from the MCS set, i.e. $\mathbf{r}=\mathbf{d}^{(Q)}$.

To generate MCS set many MCS algorithms can be derived. There are few promising MCS algorithms that are proposed and analyzed to produce the MCS set, i.e. symbol constellation approach, phase approach and interleaving approach. Using symbol constellation approach, the MCS set is produced such that the elements $d_n^{(p)}, n=1,2,...,N$ of $\mathbf{d}^{(p)}$ belongs to the same constellation as elements of original sequence. In symbol constellation approach the P index vectors are defined, the MCS vectors $\mathbf{d}^{(p)}, p=1,2,...,P$ are obtained by taking the symbols from the constellation space according to the defined vectors. In the phase approach, the random phase shifts are applied to the original data symbols to produce the MCS set. In the interleaving approach, the original sequence is permuted in pseudorandom order to produce the MCS set. Above explained approaches are not the only approaches to generate MCS set. Other approaches can be developed to generate MCS set.

B. Sidelobe Suppression by SW

SW [3] sidelobe suppression unit performs the multiplication of real valued weighting factors with symbols. The vector **d** is fed into the SW sidelobe suppression unit which outputs $\mathbf{q} = (q_1, q_2, ..., q_N)^T$. Hence, the vector **q** is given by

$$q_n = g_n d_n,\ n = 1, 2, ..., N \tag{4}$$

The weighting factors $g_n, n = 1, 2, ..., N$ are selected such that the sidelobes of the transmission signal are suppressed. Finally, the weighted vector **q** is modulated on N subcarriers using the IDFT. After that, P/S conversion is performed.

As explained in [3], to minimize the sidelobes of the weighted transmission signal **q**, we have to determine the vector $\mathbf{g} = (g_1, g_2, ... g_N)^T$ by solving the following optimization problem

$$\min_{g} \|\mathbf{Sg}\|^2 \tag{5}$$

where **S** contains samples of original transmit signal in optimization range.

There are two constraints on the weighting vector **g** . The first constraint ensures that the transmission power should not change. The second constraint ensures that the elements of **g** are in between the limits. Each subcarriers transmission power is controlled through the ratio $\rho = g_{\max} / g_{\min}$. As we increase the ratio ρ , few subcarriers get small transmission power which results in performance degradation because if any subcarrier has less power at receiver it cannot be decoded properly.

C. Sidelobe Suppression by Windowing

As illustrated in Figure 2 and Figure 3, the time domain transmit signal is multiplied with windowing function. Windowing can be applied to OFDM symbols. Windowing technique can be used to suppress the sidelobes in OFDM system. A well known window is the raised cosine window [7], which can be defined as

$$w^{RC}(t) = \begin{cases} 0.5 + 0.5\cos\left(\pi + \dfrac{\pi t}{\alpha T_{rc}}\right) & 0 \le t < \alpha T_{rc} \\ 1.0 & \alpha T_{rc} \le t < T_{rc} \\ 0.5 + 0.5\cos\left(\dfrac{\pi (t - T_{rc})}{\alpha T_{rc}}\right) & T_{rc} \le t < (1+\alpha) T_{rc} \\ 0 & else \end{cases} \tag{6}$$

where α, 0 ≤ α ≤ 1 denotes roll-off factor. The symbol duration is equals to

$$T_{rc} = (T_s + T_{prefix} + T_{postfix}) / (1 + \alpha) \tag{7}$$

After applying windowing, the time structure of OFDM signal is shown in Figure 5. The length of segment has been enlarged by prefix T_{prefix} and postfix $T_{postfix}$. The length of prefix covers the roll-off region and guard time i.e., $T_{prefix} = \alpha T_{rc} + T_G$ and the length of postfix only covers the roll-off region i.e., $T_{postfix} = \alpha T_{rc}$. After windowing the transmit signal, the sidelobes in OFDM system can be significantly reduced. By taking Fourier transform of Equation (6), the spectrum of single subcarrier of the windowed transmit signal is equal to

$$s_n^{rc}(f) = \mathbf{si}(\pi f T_{rc}).\frac{\cos(\alpha\pi f T_{rc})}{1 - (2\alpha f T_{rc})^2} \tag{8}$$

The samples of original transmit signal in the optimization range have to be determined according to Equation (8).

IV. NUMERICAL RESULTS

Numerical results illustrate the effectiveness of the combination of SW with windowing concept and combination of MCS with windowing.

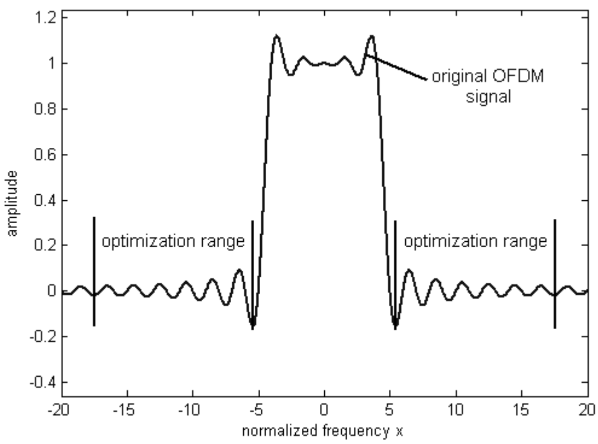

Fig.4. Block diagram of the optimization range and OFDM signal in frequency domain.

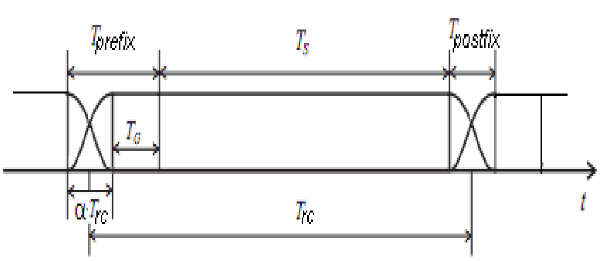

Fig.5. OFDM cyclic extension and windowing.

Binary phase shift keying modulation is applied and no channel coding is considered. The number of used subcarriers is set to N = 16. The spectra of the OFDM signals with combination of SW with windowing and without combination of SW with windowing are illustrated in Figure 6. Here the ratio ρ for SW is set to 2. The roll-off factor is set to $\alpha = 0.2$. The SW with windowing reduces OFDM sidelobes by more than 28 dB. If the ratio ρ for SW is increased then even higher sidelobe suppression results can be achieved but it degrades the system performance.

The spectra of the OFDM signals with combination of MCS with windowing and without combination of MCS with windowing are illustrated in Figure 7. Here we use the MCS set of P = 4 to generate MCS sequences. The roll-off factor is set to $\alpha = 0.2$. The MCS with windowing reduces OFDM sidelobes by more than 26 dB. If the MCS set size is increased then even higher sidelobe suppression results can be achieved but it degrades the system performance.

In Figure 8, OFDM systems with combination of MCS with windowing, and combination of SW with windowing are compared by using bit error rate (BER) versus signal-to-noise ratio (SNR) curves. OFDM system with combination of SW with windowing has more SNR degradation as compare to OFDM system with combination of MCS and windowing. For BER versus SNR, we use the MCS set size of p = 4, the value of ρ is set to 2 and roll-off factor is set to $\alpha = 0.2$.

V. CONCLUSION

We combined SW technique with windowing and MCS with windowing to suppress the sidelobes of OFDM transmission signal. By combining SW with windowing and MCS with windowing the spectral efficiency of OFDM based transmission systems can be improved and this approach can be applied to OFDM based overlay system to avoid interference towards the legacy system sharing the same frequency band. SW with windowing reduces OFDM sidelobes by more than 28 dB and MCS with windowing reduces OFDM sidelobes by more than 26 dB.

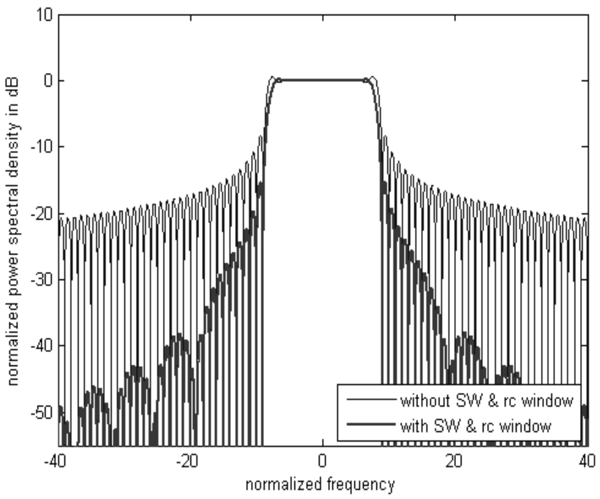

Fig.6. OFDM spectrum with SW and raised cosine window.

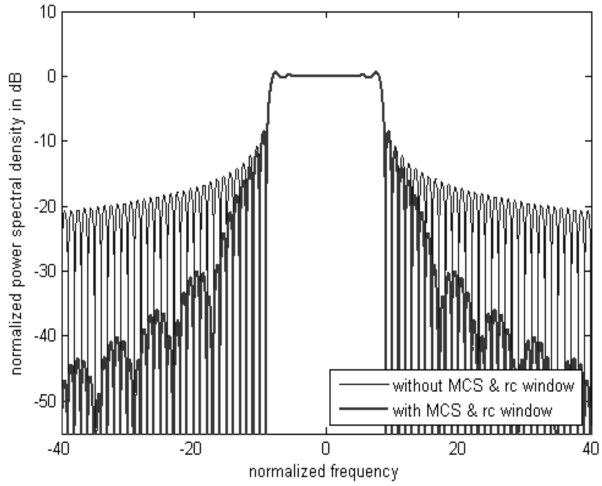

Fig.7. OFDM spectrum with MCS and raised cosine window.

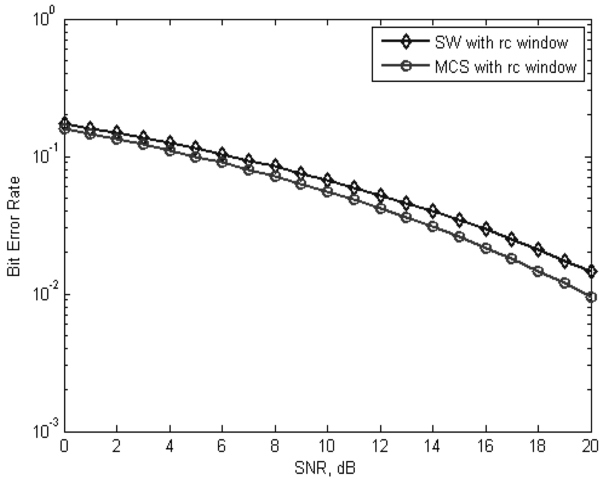

Fig.8. BER versus SNR for OFDM system with combination of MCS and windowing, and OFDM system with combination of SW and windowing.

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