# A Robust Controller and its Optimization for a Nonlinear Dynamical System

Nasimullah, Muhammad Irfan Khattak, Muhammad Shafi, and Naeem Khan

***Abstract*— This paper formulates a robust controller for a nonlinear dynamical system which is subjected to external disturbances and nonlinear friction with adaptive fuzzy logic compensator. Adaptive fuzzy system is used to estimate unknown disturbance, hysteresis phase plane, zero slip displacement and friction torque. After formulating the model, the controller is derived and closed loop stability is proved using Lyapunov function. The sliding mode control tracks the reference position command while fuzzy logic system compensates for friction and unknown disturbance torque. Parameters of controller are optimized using mixed integer optimization as first step and multi island genetic algorithm as second step. Numerical simulations are presented to prove effectiveness of the derived controller.**

***Index Terms*— Robust Controller, Optimization, Nonlinear system, Friction, Adaptive fuzzy system, Sliding mode control.**

## I. Introduction

Sliding mode control method is model based approach which is robust against system uncertainties with known upper bounds. Limitation of classical sliding mode control is high frequency chattering [1, 2]. A lot of literature discussed chattering minimization problem and different methods are incorporated for it. Disturbance observer based sliding mode control is proposed by authors of [3] in which the unknown disturbance responsible for excitation of chattering is compensated using an observer based method [3]. In recent years combining artificial intelligent methods with sliding mode control is of great interests for research community. To estimate unknown disturbances and minimize chattering problem, adaptive fuzzy sliding mode control methods are proposed for nonlinear systems and robot manipulators in [4, 5, 6, 7, and 8]. Friction is a nonlinear phenomenon in nature and its true mathematical representation is not possible to establish [9]. Several empirical relations are established based on experimental findings. These empirical models include static and dynamics friction models [10, 11, and 12]. Although dynamic friction models capture friction phenomena efficiently but its major limitations include parameters identification in offline experiments.

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Any uncertainty in the experiment would lead uncertainty in the identified parameters [13]. Model free methods do not require model parameters. A lot of literature reports adaptive fuzzy system for friction estimation [14-16] and its efficiency is proved experimentally. Authors of [17] proposed online disturbance compensation scheme for DC motor. A controller with optimum parameters can achieve best control performance. It is tedious to tune controller parameters using conventional methods or by hit and trail methods. Particle swarm optimization (PSO) method is easily implementable and it requires low computational resources. PSO optimization is proposed for proportional, integral and derivative (PID) controller formulated for automatic voltage regulator (AVR) system [18]. Several other popular techniques including genetic algorithm, simulated annealing, tabu search and neural networks are discussed in details by authors of [19]. Genetic algorithm (GA) is used to optimize controller parameters for electro hydraulic system [19] and modified GA is proposed for best performance PID controller [20]. Modified bacteria forging algorithm is used to optimize controller of nonlinear system [21].

## II. Organization of Paper

Based on the above literature survey, this article formulates adaptive fuzzy robust controller for nonlinear dynamical system and stability of the controller is proved using Lyapunov function. Parameters of controller are optimized using two step optimization with mixed integer optimization as first step and multi island genetic algorithm as second step. Numerical results are presented to show effectiveness of the proposed controller.

## III. Problem Formulation And Controller Derivation

Consider the following second order nonlinear system as

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = A(X,t) + B(X,t)u + \beta(X,t) \\ y = x_1 \end{array} \right\} \tag{1}$$

Here state vector is $X(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $u$ is the control input, $\beta(X,t)$ is the external disturbance, $A(X,t) = A_n(X,t) + \Delta A(X,t)$, $\Delta A(X,t)$ is the parametric uncertainty, $B(X,t) = B_n(X,t) + \Delta B(X,t)$, $B_n(X,t) > 0$ is the nominal input term and $\Delta B(X,t)$ is the input parametric uncertainty.

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**Assumption 1:** Nominal parameters of the system are known and state vector is available to formulate control law.

**Assumption 2:** $\psi(X,t)$ is called the lumped uncertainty and defined as

$$\psi(X,t) = \Delta A(X,t) + \Delta B(X,t)u + \beta(X,t) \tag{2}$$

With reference signal vector $X_d = \begin{bmatrix} x_d & \dot{x}_d \end{bmatrix}^T$, tracking error vector is defined as

$$\begin{Bmatrix} E_1 = x_1 - x_d \\ E_2 = x_2 - x_d \end{Bmatrix} \tag{3}$$

Sliding surface vector is defined as

$$\begin{Bmatrix} S_1 = E_2 + CE_1 \\ S_1 = E_2 + CE_2 \end{Bmatrix} \tag{4}$$

Here $C$ represents a constant parameter greater than zero. Combining Eq. 1, 3 and 4 yields;

$$S_1 = A_n(X,t) + B_n(X,t)u + \psi(X,t) - x_d + CE_2 \tag{5}$$

Letting $S_1 = -KS_1 - eq.\text{sgn}(S_1)$[1, 2], the proposed robust controller is formulated as;

$$\begin{aligned} u = B_n(X,t)^{-1}[-A_n(X,t) + \ddot{x}_d - CE_2 - KS_1 \\ -eq.\text{sgn}(S_1)] \end{aligned} \tag{6}$$

Here reaching law gain vector is $[K \;\; eq]$. The controller derived in Eq. 6 is robust to uncertain term $\psi(X,t)$ provided that; $K$ and $eq > \psi(X,t)_{\max}$ .It means that upper bound of uncertainty term should be exactly known. The disadvantage of the controller derived in Eq. 6 is high frequency chattering because gain of robust controller $[K \;\; eq]$ should be large enough to compensate the uncertainty term. To solve the problem; fuzzy logic system is introduced which is used to estimate the uncertain term $\psi(X,t)$. So Eq. 6 is modified as;

$$\begin{aligned} u = B_n(X,t)^{-1}[-A_n(X,t) + \ddot{x}_d - \tilde{\psi}(X,t) - CE_2 \\ -KS_1 - eq.\text{sgn}(S_1)] \end{aligned} \tag{7}$$

Lyapunov function is [5, 6, 7, and 8];

$$\begin{Bmatrix} V = \dfrac{1}{2}{S_1}^2 + \dfrac{1}{2\eta_i}\displaystyle\sum_{i=1}^{n}\tilde{\theta}_i^2 \\ \dot{V} = S_1\dot{S}_1 + {\eta_i}^{-1}\displaystyle\sum_{i=1}^{n}\tilde{\theta}_i\dot{\tilde{\theta}}_i \end{Bmatrix}$$

Here $\theta_i^c$ represents unknown function to be estimated. To estimate the unknown function fuzzy system is used and defined as [5];

$$\begin{cases} y_j = \dfrac{\sum_{l=1}^{M}\bar{y}_j^l\left(\prod_{j=1}^{n}\mu_{A_i^l}(x_i)\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)}, \; j = 1,2,...,m \\ y_j = \sum_{l=1}^{M}\bar{y}_j^l\xi(x) = \Theta_j^T\xi(x), \; j = 1,2,...,m \\ \xi(x) = \dfrac{\prod_{i=1}^{n}\mu_{A_i^l}(x_i)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)}, l = 1,2,...,M \end{cases} \tag{9}$$

Combine Eq. 5, 7 and 8;

$$\begin{aligned} \dot{V} = S_1[\psi(X,t) - \tilde{\psi}(X,t) - KS_1 - eq.\text{sgn}(S_1)] \\ + {\eta_i}^{-1}\sum_{i=1}^{n}\tilde{\theta}_i\dot{\tilde{\theta}}_i \end{aligned} \tag{10}$$

Define fuzzy error $\varepsilon_f$ as [5, 6]

$$\begin{aligned} \varepsilon_f &= \psi(X,t) - \hat{\psi}(X,t) \\ \tilde{\theta}_i\xi_i(\dot{\theta}_i) &= \hat{\psi}(X,t) - \tilde{\psi}(X,t) \end{aligned} \tag{11}$$

Combine Eq. 10 and 11, one obtains;

$$\dot{V} = S_1\tilde{\theta}_i\xi_i(\dot{\theta}_i) + {\eta_i}^{-1}\sum_{i=1}^{n}\tilde{\theta}_i\dot{\tilde{\theta}}_i - S_1\varepsilon_f - K{S_1}^2 - eq.|S_1| \tag{12}$$

From Eq. 12;

$$\begin{Bmatrix} -S_1\varepsilon_f \le \delta_1 \\ \dot{\tilde{\theta}} = -\eta_i S_1\xi_i(\dot{\theta}_i) \end{Bmatrix} \tag{13}$$

Second term of Eq. 13 represents adaptive fuzzy system which is used for estimation of lumped uncertainty $\psi(X,t)$ . The first row of Eq. 13 represents upper bound of fuzzy approximation error. For this case gain of robust controller $[K \;\; eq] > \delta_{1\max}$ and $\delta_{1\max} < \psi(X,t)_{\max}$. Gain of robust controller is less with adaptive fuzzy compensator as compared to non-adaptive case. Hence Eq. 7 offers minimum chattering. Combining Eq. 12 and 11 yields;

(14)

From Eq. 14, it is concluded that sliding condition is reached and system states will remain on sliding surface. Block diagram of proposed control method is shown in Figure 1.

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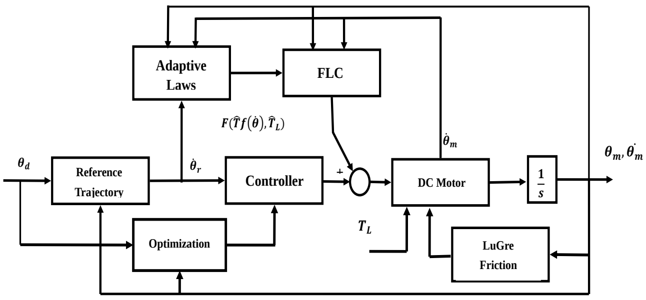


Fig. 1 Controller block diagram

### A. *Parameters Optimization*

Parameters of controller are optimized using two step optimization method. Mixed integer optimization is used as first step and multi island genetic algorithm as second optimization step. Mixed integer optimization use sequential quadratic programming to locate initial peak. If all the parameters are not real the program will search nearest points that satisfy integer value limits of non real parameters, new constraints are set and the re-optimization is performed [22]. Multi Island GA not only focuses on local but also to find global optimum points in the parameter space. In Multi Island GA each population of individual is divided into many sub population called island [23] .All traditional GA are performed on all sub population. Some individual are then selected from each island and migrated to different islands periodically, this process is called migration which is controlled by two parameters. These two parameters calculates migration interval and rate. The objective function is given by $F = w_1 \sum_{t=0}^{T} |e(t)| + w_2 \sum_{t=0}^{T} |u(t)|$ where $w_1$ and $w_2$ are the weights of tracking error and chattering.

## IV. Results And Discussion

For simulation analysis, the proposed controller is optimized for DC motor position control system which is subjected to nonlinear friction, external disturbance and unmolded dynamics. The mathematics model of DC motor plant can be described as follows []:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -Ax_2 + Bu + \psi(X,t) + T_f \end{cases} \tag{14}$$

Here the state variables $x_1$ and $x_2$ are the angle and velocity of the DC motor system, parameters $A = \dfrac{K_t K_b}{JR}$, $B = \dfrac{K_t}{JR} + \dfrac{B}{J}$, $u$ is the control effort, $\psi(X,t) = 10\sin t + 0.2x_1 + 3x_2^2$; represents lumped uncertainty including parametric uncertainty and external disturbance, $T_f$ is the nonlinear friction torque parameters of which is given in Table 1.

TABLE I
SYSTEM PARAMETERS

| Symbol | Quantity | Values |
|---|---|---|
| $B$ | Viscous coefficient | 0.244 Nm.s/rad |
| $J$ | Inertia | 0.004 Kg. m^2 |
| $K_t$ | Torque constant | 5.732 N.m/A |
| $K_b$ | Backemf constant | 3.6 N.m/V |
| $R$ | Winding resistance | 7.5Ω |
| $L$ | Winding inductance | 1mH |
| $T_S$ | Static friction | 3N.m |
| $T_C$ | Coulomb friction | 2.7N.m |
| $a_0$ $a_1$ $a_2$ | Friction coefficient | 200, 2.5, 0.02 |

### A. *Parameters Optimization Simulation*

Reference angular command of the system is $\theta_r = 0.1 * Sin(t)$. Initially controller parameters are set as; $[K \quad eq \quad C] = [1 \quad 0.1 \quad 2]$ and learning rate for adaptive fuzzy system $\eta_i = [0.001, 0.002]$. Weights of objective function are set as $[w_1 \quad w_2] = [0.8 \quad 0.2]$. Figure 2 shows controller parameter convergence against run counter. After convergence, optimized parameters of controller are $[K \quad eq \quad C] = [9.5 \quad 1.8 \quad 15]$. Figure 3 shows convergence of objective function against run counter. The objective function converges as run counter reaches around 500 counts.

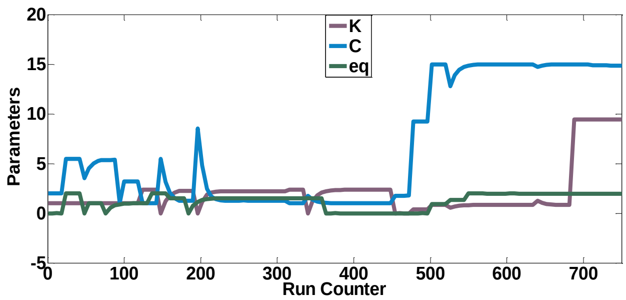


Fig. 2 Estimated parameters VS run counter

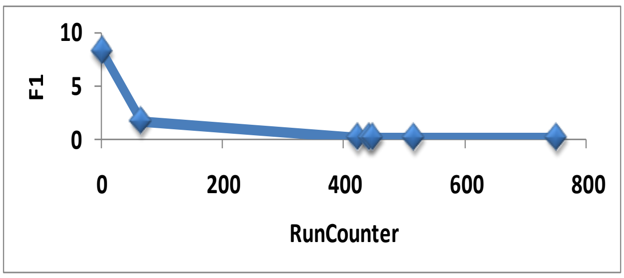


Fig. 3 Objective function VS run counter

### B. *Step Response Comparison*

Reference angular command of system is a step type having magnitude of 1 radian. Step response of optimized SMC is compared with PID and SMC without optimization. Parameters of SMC without optimization are selected as

$[K \quad eq \quad C] = [1 \quad 0.1 \quad 2]$ and for PID as $[K_p \quad K_i \quad K_d] = [10 \quad 8 \quad 1.5]$. From Figure 4, it is concluded that optimized SMC shows best response as compared to other two controllers. In case of PID large overshoots are observed while for SMC without optimization, settling time is very large.

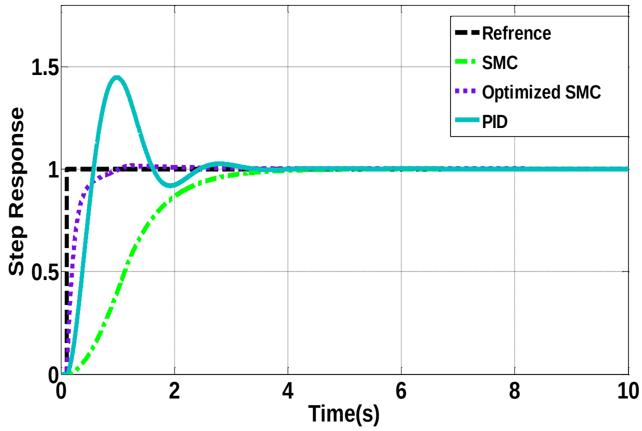


Fig. 4 Step response comparison

## C. Adaptive Fuzzy System Estimation Results

Reference angular command of the system is $\theta_r = 0.1 * Sin(t)$ radians. Parameters of friction torque acting on system are given in Table 1. Disturbance torque of $\beta(X,t) = 10\sin t$ is applied. Figure 5 compares estimated friction with model friction.

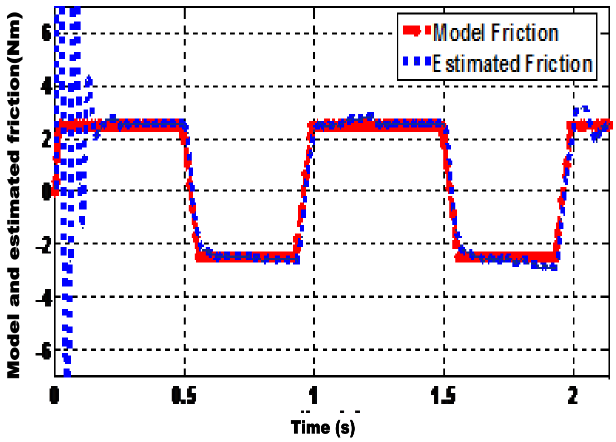


Fig. 5 Model and estimated friction torque

The Convergence error in estimated results is small except large oscillations in transient time which are acceptable. Classical friction models cannot capture friction phenomenon efficiently. In order to capture dynamic properties of friction, some dynamic friction models are introduced like Lugre model, Bliman-Sorine model and Dahl model. From Figure 6 it is concluded that the adaptive fuzzy system captures hysteresis phase plane as accurately as Lugre model and the estimation error is bounded.

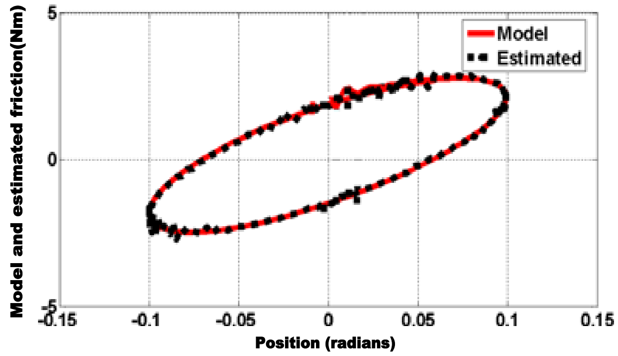


Fig. 6 Hysteresis phase plane simulations

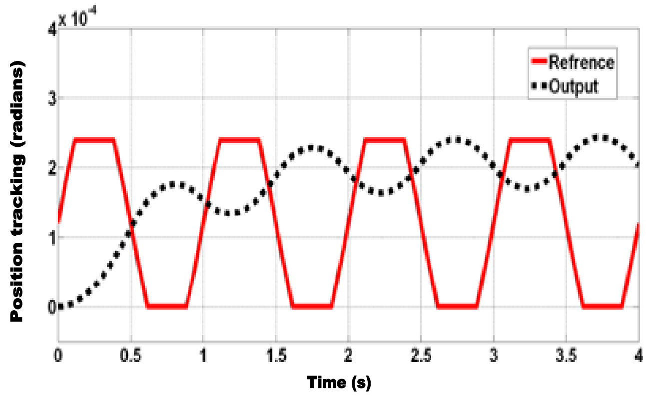


Fig. 7 Zero slips displacement simulations

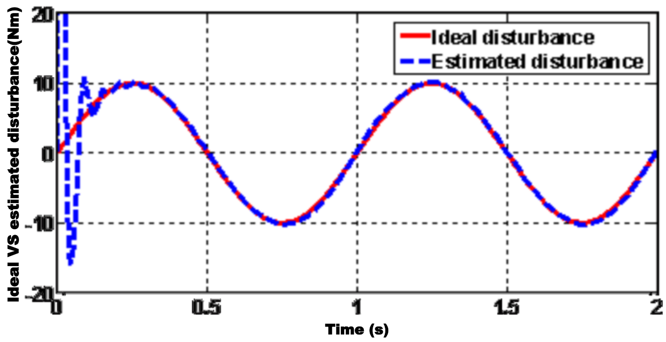


Fig. 8 Estimated disturbance torque

Zero slip displacement estimation is shown in Figure 7. For this purpose reference angular trajectory is set to $\theta_r = 2.5 * 10^{-4} * Sin(t)$. The simulation results confirm that fuzzy output accurately estimates the zero slip friction disturbances. The estimated disturbance torque simulations are presented in Figure 8. The estimation error is bounded and less than 1%.

## D. Position Tracking Simulations

Reference command of the system is $\theta_r = 0.1 * Sin(t)$. Optimized parameters are set as $[K \quad eq \quad C] = [9.5 \quad 1.8 \quad 15]$ and learning rate for adaptive fuzzy system is given as $\eta_i = [0.001, 0.002]$.


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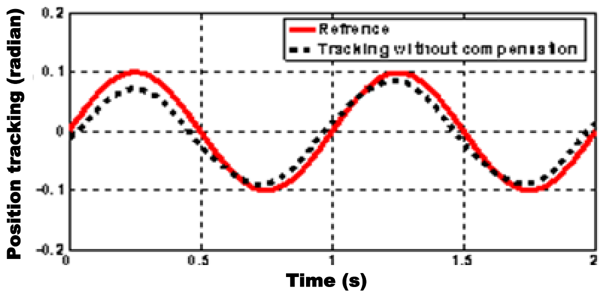


Fig. 9 Position tracking without compensation

Figure 9 shows tracking performance without fuzzy compensation. Tracking error is about 20%. Tracking error is big as gain of switching controller is small. By increasing gain, tracking error can be compensated but at the cost of high frequency chattering in control signal.

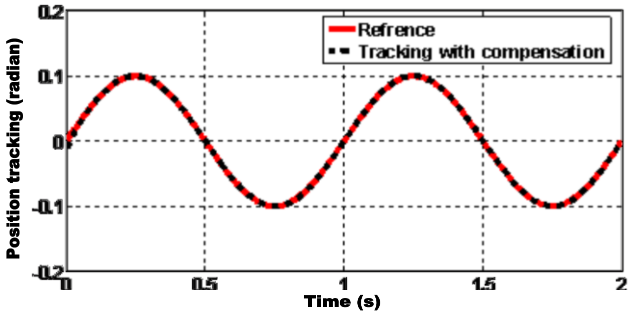


Fig. 10 Position tracking with compensation

Figure 10 shows position tracking performance with fuzzy compensation. Position tracking error is less than 1%. In this case, position tracking error is small enough with smallest gain of switching controller. Thus minimum tracking error is achieved with minimum chattering of control signal as shown in Figure 11.

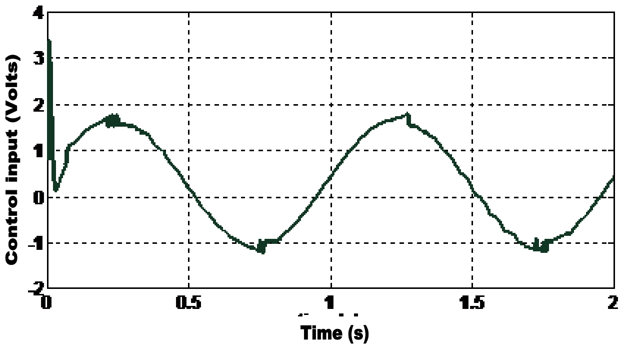


Fig. 11 Control signal simulations

## V. Conclusions

A robust controller is formulated for a class of nonlinear dynamical system which is subjected to nonlinear friction, external disturbance and unmolded dynamics. Parameters of controllers are optimized using two step optimization. Adaptive fuzzy system is used to estimate and capture dynamic behavior of friction phenomena and nonlinear effects. Optimized SMC offers best step response performance. Moreover, with optimized controller and adaptive fuzzy system, both tracking error and chattering phenomena reduces significantly.

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