Higher Order MIMO System Capacity Assessment using Singular Value Decomposition while considering Power constraint problem

A.M Soomro, M.A Shah

Abstract – **Wireless communication technology has developed many folds over the past few years. One of the techniques to enhance the data transfer rates is called Multiple Input Multiple Output (MIMO), in which multiple antennas are employed both at transmitters and the receivers. Multiple signals are transmitted from various antennas at transmitter site using the same frequency and separated in space. One of the promising features of this technology is to exploit multipath fading which is one of the main obstacles for SISO (Single Input Single Output). Apart from multipath fading, it provides a great range of signal transmission and high data rates without increasing the available bandwidth and power, which are the precious resources for communication system. In this paper, channel capacity for higher order MIMO systems of the order from 2x2 to 8x8 has been assessed. Singular Value Decomposition (SVD) has been mathematically discussed for resolution of Channel Matrix (H). MATLAB has been used for simulating the results. While determining the capacity, power constraint problem is considered which is handled by water-filling algorithm efficiently. Higher spectral efficiency has been achieved by using higher system order.**

Index Terms – **MIMO, SVD, Noise Variance, PDF, Water Filling Algorithm, Power Constraint.**

I. INTRODUCTION

MIMO (Multiple Input Multiple Output) is an efficient technology to be considered for LTE Advanced, HSPA+, WiMax, and 802.20, 802.11n/ac/ad and other 4G and 5G standards which are still under research. The purpose of MIMO is to enhance the data transfer rates which are mostly demanded by the bandwidth hungry applications like interactive video communication [1, 2]. This is accomplished with the help of Spatial Multiplexing [3]. In this technique it breaks the input data stream into parallel streams which are multiplexed at the receiver causing a drastic increase in throughput. It also increases the reach of the signal a means of beam-forming and transmitting same power quantity [4].

Integrity of data is also managed efficiently as MIMO exploits the Multipath fading effects unlike SISO which tries to overcome such effects. Due to major increment in MIMO systems' capacity, it has become inevitable to assess the capacity and suggest different pragmatic solutions to improve the transmission of error-free data. There are different ways through which the channel capacity of MIMO system can also be estimated. In this paper Singular Value Decomposition is used to decompose the Channel matrix (H). Furthermore, MIMO channel capacity is assessed and Probability Density Function (PDF) of elements in decomposition matrix have been calculated and then analyzed up to the system order of 8x8. The work has been accomplished by simulating obtained results on MATLAB.

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A. MIMO System Model

The simplest MIMO system model can be deduced from Fig.1 and can be mathematically represented as,

$$y = Hx + n \tag{1}$$

Here, H represents the MIMO channel matrix of order; this contains the elements denoting the possible logical paths between N transmitter and M Receiver antennas, is the gain of each individual path. Equation (2) gives matrix of MIMO system with order.

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1j} \\ h_{21} & h_{22} & \cdots & h_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ h_{i1} & h_{i2} & \cdots & h_{ij} \end{bmatrix} \in C^{N_R \times N_T} \tag{2}$$

Fig.1 describes the channels which are created by implementing multiple antennas at both receiver as well as transmitter. These channels can be considered as parallel pipes that are responsible for carrying data.

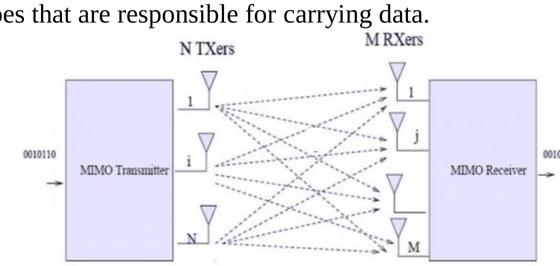

Fig.1. MIMO System Model

B. MIMO Channel Modeling

MIMO technology involves multiple antennas at the transmitter and receiver that undergo a complex channel modeling and demands good knowledge about Path loss, Doppler and Delay spread [2, 6]. To tackle this problem, both antenna correlation at the transmitter and receiver, and the shadowing are taken into considerations. The channel matrix in MIMO consists of either Line of Sight (LOS) or Non Line of Sight components (NLOS) as both may be present in radio channel [7]. In this case H can be modeled as the sum of

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H_{LOS} and $H_{N\text{-}LOS}$.

$$H = H_{LOS} + H_{N\text{-}LOS} \tag{3}$$

Strong H_{LOS} in equation (3) depicts that Channel has higher Rician factor.

II. SINGULAR VALUE DECOMPOSITION

SVD is a technique which is capable enough to mitigate co-channel interference by means of dividing MIMO channel into non-correlated SISO channels. Apart from this, it is also able to provide power allocation mechanism which enables us to use the full capacity utilization of MIMO system [8, 9]. Singular Value Decomposition (SVD) is the technique in which the channel matrix is subdivided into three special characteristic matrices $(U, \sum, V^H)$. Where U and V are known as unitary matrices. $(.)^H$ is the conjugate transpose of matrix as baseband symbols sent at the transmitter. While $\sum$ is $diag(\sigma_1, \sigma_2, ...\sigma_r)$. These diagonal values are placed in descending order i-e $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r > 0$ and known as channel singular values [9, 10].

A. Spatial Multiplexing

Spatial multiplexing is a type of multiplexing where distinct data bits or signals are sent through several spatial (communication) channels [11]. This is achieved by placing multiple antennas at transmitter as well as receiver site; this way increases data transmission rate which is in direct proportionality with the number of antennas placed for transmission and receiving purposes. The equation 1 represents a general MIMO model. This is a basic building block for comprehending high order system.

Where "Y" is the received vector, "X" is the transmitted vector, "H" is the channel matrix and "n" is the noise vector at the receiver side. "n" is zero for ideal system. Through SVD decomposition, channel matrix can be decomposed as given in equation 5.

$$H = U\Sigma V^H \tag{5}$$

The channel H is now decomposed in three different unique but featured matrices. The received signal can now be further analyzed by equation 6.

$$Y = U\Sigma V^H x + n \tag{6}$$

U^H has been multiplied with received signal. This gives the concept of pre-coding, which is done at the transmitter.

$$U^H Y = \tilde{Y} = U^H Y(U\Sigma V^H X + n) \tag{7}$$

$$Y = \Sigma V^H X + U^H n \tag{8}$$

The term $U^H n$ is known as modified noise vector. $\tilde{n}$ is used as its notation in this paper. Whereas transmitted vector V^H is denoted by X. After these major substitutions, equation 8 will be,

$$\tilde{Y} = \Sigma V^H V \tilde{x} + \tilde{n} \tag{9}$$

$$V^H V = U^H U = 1 \tag{10}$$

$$\tilde{Y} = \Sigma \tilde{x} + \tilde{n} \tag{11}$$

In equation 11 '$\sum$' is a diagonal matrix which is further shown in equation 12.

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \vdots \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \vdots \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \vdots \\ \tilde{n}_t \end{bmatrix} \tag{12}$$

If there are eight antennas at the receiver it shows that each signal will be received by each antenna. But due to this $\sum$ matrix, which is a decoupling matrix, separated each data pipe for separate antenna. This concept is known as parallelization of MIMO channel. That has been shown by equation 13.

$$\begin{aligned} \tilde{Y}_1 &= \sigma_1 \tilde{x}_1 + \tilde{n}_1 \\ \tilde{Y}_2 &= \sigma_2 \tilde{x}_2 + \tilde{n}_2 \\ &\vdots \\ \tilde{Y}_t &= \sigma_t \tilde{x}_t + \tilde{n}_t \end{aligned} \tag{13}$$

This shows the collection of t parallel channels, transmitting t information symbols hence also termed as Spatial Multiplexing.

B. Variance of Modified noise

Variance of modified noise $\tilde{n}$ gives good insight at the receiver site. Derivation has been discussed from equation 14 to equation 18. While equation 19 gives us a better comprehension that variance of noise at the transmitter and receiver does not change. This is because we have considered the uncorrelated channels in MIMO system as shown in equation 14.

$$\tilde{\boldsymbol{n}} = U^H n \tag{14}$$

$$\boldsymbol{E}\left\{\tilde{\boldsymbol{n}}\tilde{\boldsymbol{n}}^{\boldsymbol{H}}\right\} = \boldsymbol{E}\left\{U^H n \tilde{\boldsymbol{n}}^{\boldsymbol{H}} \boldsymbol{U}\right\} \tag{15}$$

$$= \boldsymbol{U}^{\boldsymbol{U}} \boldsymbol{\sigma}_{\boldsymbol{n}}^2 U \tag{16}$$

$$= \boldsymbol{\sigma}_{\boldsymbol{n}}^2 \boldsymbol{U}^{\boldsymbol{H}} \boldsymbol{U} \tag{17}$$

$$= \boldsymbol{\sigma}_{\boldsymbol{n}}^2 \boldsymbol{I}_{\boldsymbol{t}} \tag{18}$$

$$\boldsymbol{\sigma}_{\tilde{\boldsymbol{n}}=}^2 \boldsymbol{\sigma}_{\boldsymbol{n}}^2 \tag{19}$$

Here SNR (Signal to Noise Ratio) of Parallel Channel is,

$$SNR_i = \frac{\boldsymbol{\sigma}_{\boldsymbol{i}}^2 \boldsymbol{P}_{\boldsymbol{i}}}{\boldsymbol{\sigma}_{\boldsymbol{n}}^2} \tag{20}$$

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Here P_i is the power for the channel capacity shown in equation 25 and σ_i is the gain of i^{th} channel. A concrete concept of spatial multiplexing is given in Fig.2.

$$C_i = Log_2\left(1+\frac{\sigma_i^2 P_i}{\sigma_n^2}\right) \quad (21)$$

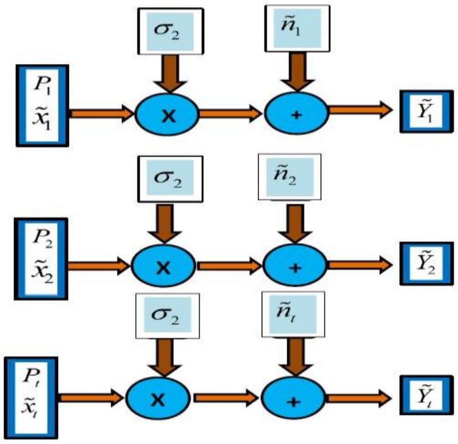

Fig.2. Spatial Multiplexing

III. MIMO CAPACITY AND ITERATIVE POWER ALLOCATION

Equation 22 describes the expansion of equation 21. Each path will have its own capacity. Greater the number of antennas, the great will be the paths leading to higher data rate.

$$C_1 = log_2\left(1+\frac{\sigma_1^2 P_1}{\sigma_n^2}\right)$$
$$C_2 = log_2\left(1+\frac{\sigma_2^2 P_2}{\sigma_n^2}\right)$$
$$\vdots$$
$$C_t = log_2\left(1+\frac{\sigma_t^2 P_t}{\sigma_n^2}\right) \quad (22)$$

Net capacity of MIMO system will be equal to the sum of capacities of individual channels.

$$C_{MIMO} = \sum_{i=1}^{t} log_2\left(1+\frac{\sigma_i^2 P_i}{\sigma_n^2}\right) \quad (23)$$

Capacity depends upon the power given to individual antennas. However, there is the limit that sum of individual antenna powers should be equal to or less than the total transmitted power as discussed in [6].

$$P_1 + P_2 + P_3 + - - - - - + P_t \le P \quad (24)$$

This gives rise to another important field of study known as Optimum MIMO channel Power Allocation or Constraint Maximization Problem. The capacity can be maximized based on the constraint given in equation 25. There is the need to calculate maximum possible capacity that proposed system can offer.

$$\sum_{i=1}^{t} P_i \le P \quad (25)$$

$$C = Max \sum_{i=1}^{t} log_2\left(1+\frac{\sigma_i^2 P_i}{\sigma_n^2}\right) \quad (26)$$

To find the minima or maxima of the function, the derivative is simply taken and made equal it to zero, however here constraint issue has been taken into considerations, therefore the use of Lagrange Multiplier is necessarily used here. This is also suggested in [5].

$$F = \sum_{i=1}^{t} \log_2\left(1+\frac{\sigma_i^2 \mathrm{P}_i}{\sigma_n^2}\right) + \lambda\left(P - \Sigma P_i\right) \quad (27)$$

The derivative is taken with respect to P_1 and made equal to zero. From equation 28 to equation 30, derivation regarding power of individual path has been discussed.

$$\frac{dF}{dP_1} = 0 \quad (28)$$

$$\frac{\dfrac{\sigma_1^2}{\sigma_n^2}}{1+\dfrac{P_1\sigma_1^2}{\sigma_n^2}} + \lambda(-1) = 0 \quad (29)$$

$$\frac{1}{\lambda} = \frac{\sigma_n^2}{\sigma_1^2} + P_1 \quad (30)$$

The power allocation among individual channels has been shown by equation 31.

$$P_1 = \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_1^2}\right)^+$$
$$P_2 = \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_2^2}\right)^+$$
$$\vdots$$
$$P_t = \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_t^2}\right)^+ \quad (31)$$

$$\sum_{i=1}^{t} \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_t^2}\right)^+ = P \quad (32)$$

$$x = \begin{cases} x \ \ if \ \ x > 0 \\ 0 \ if \ \ x \le 0 \end{cases} \quad (33)$$

Power constraint bounds the system to follow that sum of power of individual channels must be less than or equal to total power fed into transmission system.

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$$\sum_{i=1}^{t} P_i = P \tag{34}$$

$$\sum_{i=1}^{t}\left(\frac{1}{\lambda}-\frac{\sigma_n^2}{\sigma_t^2}\right)^{+} = P \tag{35}$$

Equation 39 gives a mathematical way of allocating power among parallel channels. This has solved the power allocation problem. Through this, each channel can be efficiently given power while considering power constraint. This is an iterative fashion of distributing power based on its working termed as water filling algorithm. For performing simulations, MATLAB's built-in command, *[Capacity (i,j) PowerAllo] = WaterFilling_alg(Pt,* λ *(:,j),B,N0)*. Here P_t is total power fed to transmitting system. This could also be done by using this equation directly with the help of loop.

IV. RESULTS AND DISCUSSIONS

In this section, simulation results have been presented, which are carried out in MATLAB. The MIMO channel capacity and the PDF of the matrix (λ) in SVD Decomposition of matrix H have been analyzed, considering the power constraint. The system up to 8th order is considered. In obtaining the results, highly scattered environment is taken into considerations. The water filling algorithm is used for the efficient power allocation in parallel channels after decomposition of the channel matrix.

There are only two antennas in Single Input Single Output (SISO) communication system. Owing to this, multipath fading proves to be a problem as it causes multiple copies of the original signal. This limits the system's capacity in terms of spectral efficiency. MIMO is designed not only to combat this obstacle but also exploits it with the means of spatial multiplexing. Fig.3 shows a greater difference in system's overall capacity between SISO and MIMO. The capacity for SISO and MIMO systems are shown and compared. It can be deduced that capacity is increases as the number of antennas is increases. The black line is showing the capacity of SISO system and the red, green and yellow lines show the system capacity for 2x2, 3x3 and 4x4 MIMO systems respectively. The MIMO system shows the maximum capacity theoretically which has been proved by Fig.3. Also as the number of transmit and receive antenna increase in MIMO system, their efficiency also increase.

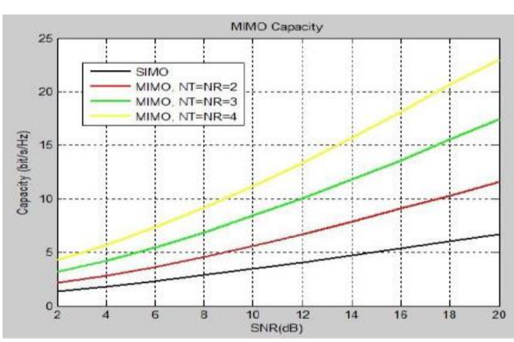

Fig.3. SISO VS MIMO System Capacity

Fig. 4 illustrates the simulation result of water filling algorithm in MIMO system. It is difficult task to allocate power in channels which are contaminated by noise. In such case Water Filling Algorithm proves to be an efficient and fast technique. For its implementation input transmitted power, number of noisy channels and noise amounts per channel have been pre-defined.

The results suggest that noisy channels are allocated less power as compared to less noisy. This makes the transmission immune to transmission errors.

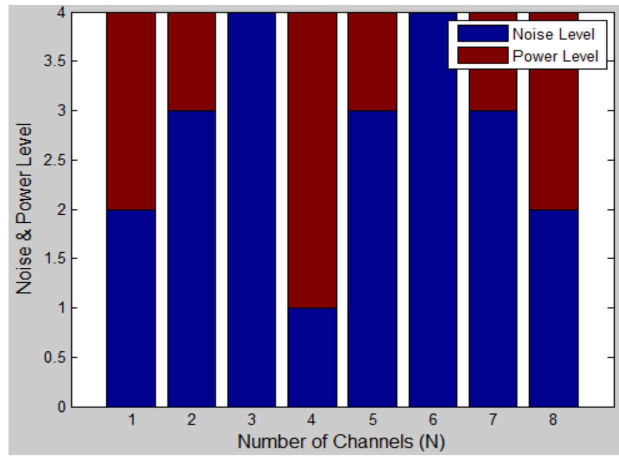

Fig.4. Water Filling Algorithm

The MIMO system that has been considered in this paper has 8 transmitting receiving antenna. The results in Fig.5 indicate that capacity of system increases with the increase in the number of transmit and receive antennas.

The following simulation has been performed by first allocating power among strong channels as mentioned in equation 39. The process of water filling algorithm is similar to filling the water in the vessel. By using this algorithm high capacity in terms of bits/sec/Hz is achieved.

Fig.5 clearly depicts that higher spectral efficiency is proportional with the order of the MIMO system. It also shows that a SISO system has the maximum capacity of 6 bits/sec/Hz at 20 dB. Contemporary, higher number of antennas attains great capacity of 40 6 bits/sec/hz at 20 dB for 8x8 on the account of getting system towards more complexity. The capacity gradually increases with the increase of antennas at the communicating ends.

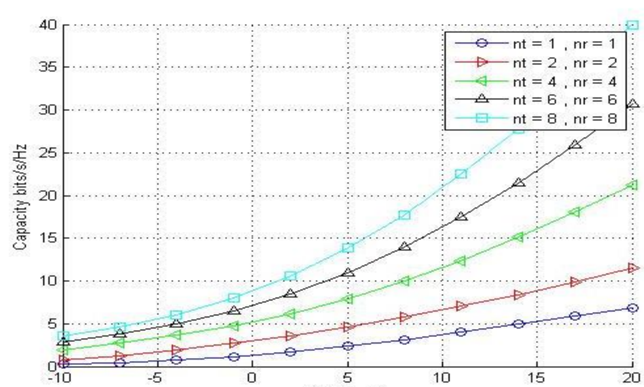

Fig.5. MIMO SYSTEM CAPACITY

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In fig.6, the PDF of elements in SVD decomposition matrix of H matrix is graphically shown. It shows that as the MIMO order increase the amplitude of the elements go for higher values as compared to the lower order. Looking into the SISO channel, the PDF acts a small-amplitude half wave sinusoidal pulse. Whereas going to the higher order, enables higher value even in the starting and lasts longer.

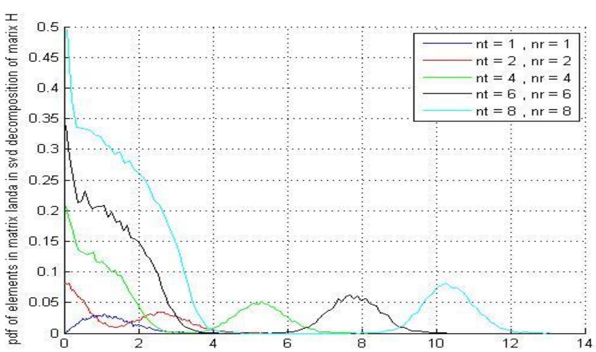

Fig.6. PDF o elements in matrix λ in SVD decomposition of matrix H

V. CONCLUSION

In this paper, MIMO system capacity and PDF of SVD-based decomposed λ have been analyzed. The both system capacity as well as PDF is calculated by changing system order up to 8x8. It is observed that system capacity is proportional with the number of antennas placed both at transmitter as well receiver. Higher system order supports higher amplitude of PDF of λ that supplements greater overall efficiency. A mathematical model for iterative power allocation has been derived which encompasses Water-Filling algorithm, where Lagrange Multiplier is also discussed. At the higher SNR value, the independent and identically distributed (i.i.d) channel capacity outperforms the correlated channel capacity. But At very low SNR value correlated channel capacity outperforms the i.i.d channel capacity. It can be seen that MIMO-OFDM system can significantly increase the channel capacity of the system with the inclusion of more antenna to the system.

Although it gives great benefit in going for higher order just like higher channel capacity and improvement in PDF of elements, yet it should not be avoided that the complexity of system will exponentially decrease. So use of some effective trade-off should be done to get optimum advantages in all issues related to MIMO technology.

VI. FUTURE RECOMMENDATIONS

MIMO channel capacity is calculated, but there need of significant contribution on the power allocation side. As the channel capacity is dependent over the power allocated to the particular channel.

The research done in this paper will prove to be a platform to work on more advanced decomposition techniques like regularized-SVD.

The greater increment in data rates of MIMO system provoke the problem of Inter-Symbol Interference (ISI) that can be mitigated by using Equalization Filters. Research in selecting the efficient filter without compromising with system capacity is also going on.

The paper focuses on simulation work, while numerical analysis of MIMO system capacity will be done in future to implement for various relevant areas of applications.

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