# Vehicle Dynamics Parameter Design Using Different Optimization Techniques 

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#### Abstract

Car Design Optimization deals with the optimal structural design of multipart engineering systems which require parameters that dealing amongst the disciplines. In this paper we used optimization techniques to optimize the aerodynamics of a car to reduce drag and drift that optimize design to reduce lap time. Our execution involves seven different design parameters that is weight scattering, aerodynamic downward force scattering, mass of vehicle, height of center of gravity, radius of track, weight scattering along width and roll stiffness coefficient. The objective is to determine car parametric coefficient that minimizes lap time while satisfying yaw balance constraint. In order to achieves that several techniques is used such as Simulated Annealing, Particle Swarm optimization technique and Genetic Algorithm. Results from different techniques is obtained in this part of research as to explore suitable technique for this type of problem. Previous work which we took as our reference paper is only considered three variables that is weight scattering, weight scattering along width and roll stiffness. While we are taking into consideration seven aspects of the car, the additional four are downward force, mass of vehicle, height of center of gravity of vehicle and radius of the track. These variables also play a vital role in the speed of the car or how quick it performs turn without going into a risky condition.


## Index Terms - Optimization, Dynamics, Vehicle Design

## I. INTRODUCTION

The car structure design optimization encompasses the several car parameter simultaneously and leading to the optimal solution of a coupled composite design parameter. It is regarded as the optimal solution of the interactions. Car structural design optimization mainly reduces the complexity by breaking down a problem into subsystems which are linked by different design parameters, functions and performances.

Latin Hyper-cube Sampling is a stratified irregular pattern that gives an effective method for examining variables from their multivariate appropriations. Another system Latin square is a square exhibit in which every line and every segment comprises of the same arrangement of passages without reiteration. LHS is an outline procedure is utilized to treat all locales of the configuration space equally.

Genetic Algorithms (GA) are versatile routines which is a coordinated irregular pursuit strategy, to locate the global optimal solution in complex multi-dimensional hunt space. At first numerous individual arrangements are haphazardly created to frame an inquiry space including the whole scope of conceivable arrangements. Every point in the hunt space

[^0]speaks to one conceivable arrangement stamped by its worth (fitness). Second generation is acquired from the hybrid and change from the chose extent of existing population [25][8].

Particle swarm optimization is motivated by reenactment of social conduct identified with bird flocking[24][14]. The essential thought for the social conduct is guiding toward the middle, match neighbors' speed and keep away from impacts. PSO develops an issue by having a population of competitor arrangements. Every particle`s movement is affected by its neighborhood best known position and it's likewise guided toward the best known position in the hunt space, which are redesigned as better positions are found by different particles. This is required to move the swarm toward the best arrangements. PSO is a Meta heuristic as it makes few or no presumptions about the issue being enhanced and can look huge spaces of competitor arrangements. Every particle monitors its directions in the arrangement space which are connected with the best arrangement (fitness) that has accomplished so far by that particle. This is called individual best, pbest. Another best esteem that is followed by the PSO is the best esteem got so far by any molecule in the area of that particle. This quality is called global best, gbest. The essential idea of PSO lies in quickening every particle toward its pbest and the gbest areas, with an irregular weighted increasing speed at every time step [24].

Simulated Annealing (SA)[3] is a non-specific probabilistic Meta heuristic for the worldwide advancement issue of finding a decent estimation to the global optimal value of a given capacity in an inquiry space. It is frequently utilized when the hunt space is discrete. The name and motivation originate from tempering in metallurgy, a system including heating and controlled cooling of a material to build the extent of its precious stones and lessen their deformities. The energy equation for the thermodynamic framework is practically equivalent to the objective function, and ground state is similar to the global minimum [23].

Previous work which we chose as our reference [1] using collaborative optimization technique to minimize the laptime or we can say that it maximizes the speed of the vehicle. They only considered three design parameter, result obtained is shown in table 5 as a comparison with the result obtained by the purposed technique.

Our goal is to explore different techniques to reduce the lap-time. To take a turn on an optimal speed is a main objective as part of this paper. We considered seven different parametric variables to optimize our result instead of three use in previous paper [1]. We used four techniques to optimize the results, LHCS, GA, SA and PSO and compare results with each other. We found that PSO obtained most optimized result in our case.

[^1]$B=$ Weight Scattering along width
$\mathrm{C}=$ Aerodynamic Downward force Scattering
$K=$ Roll Stiffness coefficient
$\mathrm{U}=$ Speed $/$ Lap
$\beta=$ Vehicle drift Angle beta
$\delta=$ Vehicle Wheel Steering Angle delta
T = Time/Lap
$\mathrm{Fx}=\quad$ Forward Force
Fy $=\quad$ Side Force
$\mathrm{Fz}=\quad$ Vertical Force
FYaw $=$ lateral moment Force
YawBal =lateral moment Balance force

## II. APPLICATION

Race car design provides a rich environment for multidisciplinary design optimization. It includes knowledge of aerodynamics, structural-mechanics, vehicle dynamics and tire friction for its configuration and analysis. Each discipline got its own expertise and control over it individual best of the vehicle.

Kasprzak and Hacker [1] used multi objective optimization (MDO) to maximize race car performance across a tracks of fixed radii i.e 400 m . During race the car faces different corners and straights, for which a set of conflicting tradeoffs exist in order to design a race car performing well across turns of a radius on a single track.

Our vehicle model depends on the fantastic bike model of Milliken, which has been extended to incorporate four individual wheels. Comparisons of movement are composed for parallel speeding up, longitudinal increasing speed, and yaw balancing. The tires, which may be diverse for front and back, are displayed utilizing plain tire information including representations of Nonlinearities, for example, load affectability and sideslip angle Saturation. Wheel burdens are computed in view of static burden, streamlined down power, and horizontal burden exchange.


Fig. 1 Race Car Parametric model [1]
Figure 1 illustrates a simplified sketch of the racecar model. There are three primary design variables: roll stiffness coefficient (K), weight scattering along length (A), weight scattering along width (B) and aerodynamic downward force scattering (C). Four design parameters are normalized between 0 and 1 .

$$
\begin{equation*}
C^{\prime}=\frac{C_{L F}}{C_{L F}+C_{L R}} \tag{1}
\end{equation*}
$$

$$
\begin{array}{r}
K^{\prime}=\frac{K_{F}}{K_{F}+K_{R}} \\
A^{\prime}=\frac{a}{a+b} \\
B=\frac{a^{\prime}}{a^{\prime}+b^{\prime}} \tag{4}
\end{array}
$$

## III. MATHEMATICAL MODEL

Formulization section involves development of the equations governing race car design. The analysis begins with calculation of lift coefficients, downward forces coefficient and other parameters. It achieves with iterative analysis to solve for sideways forces, velocity and lap time for the given design parameters. All equations in this research is being cited from[1]. Table 1 gives the design parameters for race

TABLE 1. Race Car Design Variables

| Variables | Description |
| :---: | :--- |
| A | Weight distribution along length |
| C | Aerodynamic down force distribution |
| K | Roll stiffness distribution |
| m | Mass of vehicle |
| h | Height of CG from ground |
| R | Radius of the track |
| B | Weight distribution along width |

car configurations having normalized values ranging from 0.3 to 0.6 respectively except height of center of gravity, mass of vehicle and radius of track.

Design space (i.e 0.3-0.6) is for the variables A, B, C, and K . Table 2 contains the list of parameters like the radius of the track, car weight, height of CG from the ground, lengths etc which are taken to be fixed for analysis of race car configuration and it is the same as used in [1].

Figure 2 represents the connections between parallel strengths and side-slip angles. As showed, the focal point of


Fig. 2 Race car dynamical model [1]
gravity characterizes the inception of the direction framework, and clockwise moments are positive.

TABLE 2. Race Car and Track Parameters

| Parameter <br> Description | Parameter Symbol | Value | Unit |
| :--- | :--- | :--- | :--- |
| Vehicle wheel base | L | 9.67 | Ft |
| Vehicle mass | M | 41.7 | Slug |
| Height of CG | H | 1.167 | Ft |
| Front track | tF | 5.5 | Ft |
| Rare track | tR | 5.25 | Ft |
| Frontal area | RefA | 10 | ft 2 |
| Skid pad radius | Radius | 400 | Ft |
| Drag coefficient | $\mathrm{C}_{\mathrm{D}}$ | 2.9 | - |

In observation of the streamlined downward-force scattering, C. All formulization is from previous work by different authors in this we only used different techniques to improve results.

$$
\begin{align*}
& C L F=-0.5 \times C  \tag{5}\\
& C L R=-1[5 \times(-5 \times C)] \tag{6}
\end{align*}
$$

Equation (7) calculates the weight distribution of the car, HW, where $g$ is the gravity due to earth.

$$
\begin{equation*}
H W=\frac{m \times g}{2} \tag{7}
\end{equation*}
$$

Equations (8-10) govern the coefficients for front and rear downward force, FD and RD, and aerodynamic friction force, $D$, where $\rho$ is the density of atmosphere.

$$
\begin{align*}
& F D=-\frac{\rho \times C L F \times R e f A}{2}  \tag{8}\\
& R D=-\frac{\rho \times C L R \times \text { RefA }}{2}  \tag{9}\\
& D=-\frac{\rho \times C D \times R e f A}{2} \tag{10}
\end{align*}
$$

Initialized the system before proceed to evaluate the optimal solution, Table 3 indicates the parameters initial values.

Equations (11-13) determine the aerodynamic forces, where positive quantities direct downward force. The aerodynamic force acting on the front and rear wheels is represented by FzF and FzR, respectively. Fx is an aerodynamic friction force that opposes the motion.

$$
\begin{align*}
& F z F=F D \times \text { uold }^{2}  \tag{11}\\
& F z R=R D \times \text { uold }^{2}  \tag{12}\\
& F x=D \times \text { uold }^{2} \tag{13}
\end{align*}
$$

Equation (14) shows the required track effort, FxR, which is permanently positive or it is the only in one direction and that direction is considered a positive direction.

$$
\begin{align*}
& F x R=F x+[F y F \times \sin (\alpha M a x F)] \\
& +\quad[F y R \times \sin (\alpha M a x R)] \tag{14}
\end{align*}
$$

Forward-facing and back wheel weights, LFT and LRT, are given by Equations (15 and 16).

$$
\begin{align*}
& L F T=\left(F y \times \frac{h}{t F}\right) \times K^{\prime}  \tag{15}\\
& L R T=\left(F y \times \frac{h}{t R}\right) \times\left(1-K^{\prime}\right) \tag{16}
\end{align*}
$$

TABLE 3. Initialization of Lateral Force Loop

| Parameter | Description | Initial value |
| :---: | :---: | :---: |
| FyRF | Right front wheel load | 0 |
| FyRL | Left rear wheel load | 0 |
| FyRR | Right rear wheel load | 0 |
| FyF | Lateral force front axle | 0 |
| FyR | Lateral force rear axle | 0 |
| Fy | Lateral force | 0 |
| uold | Velocity last iteration | 0 |
| $\alpha$ MaxF | Max front slip angle | 0 |
| $\alpha_{\text {MaxR }}$ | Max rear slip angle | 0 |

Equations (17-20) determine the down force on each of the four wheels. For instance, FzRF, is the down force acting on the right front wheel.

$$
\begin{align*}
& F z L F=\left[\left(1-A^{\prime}\right) \times H W\right]+L F T+\frac{F z F}{2}  \tag{17}\\
& F z R F=\left[\left(1-A^{\prime}\right) \times H W\right]-L F T+\frac{F z F}{2}  \tag{18}\\
& F z L R=\left[A^{\prime} \times H W\right]+L R T+\frac{F z R}{2}  \tag{19}\\
& F Z R R=\left[A^{\prime} \times H W\right]-L R T+\frac{F z F}{2} \tag{20}
\end{align*}
$$

Taking into account the introduced tires with arranged horizontal powers because of ordinary load and slip edge, quadratic estimate is utilized to decide most extreme slip points, and $\alpha$ MaxR, and parallel strengths, FyF and FyR, for the front and back axles.

Equations (21 and 22) check the horizontal powers on the back wheels, FyLR and FyRR, and, if required, decrease these forces because of the resistive force ellipse impact.

$$
\left\{F y L R=\left\lvert\, \begin{array}{cc}
0 & \frac{F x R}{2}>|F y L R|  \tag{21}\\
\frac{F y L R \sqrt{\left|F y L R^{2}-\left[\frac{F x R}{2}\right]^{2}\right|}}{|F y L R|} & \text { else }
\end{array}\right.\right\}
$$



Fig. 3 Pre-Sensitive Analysis Result of A, B, C, k, m, h and R respectively

$$
\left\{F y R R=\left\{\begin{array}{cc}
0 & \frac{F x R}{2}>|F y R R|  \tag{22}\\
\frac{F y R R \sqrt{\left|F y R R^{2}-\left[\frac{F x R}{2}\right]^{2}\right|}}{|F y R R|} & \text { else }
\end{array}\right\}\right.
$$

Equation (23) calculates the complete back lateral force, FyR, as a sum of sideways forces acting on each of the two back wheels.

$$
\begin{equation*}
F y R=F y L R+F y R R \tag{23}
\end{equation*}
$$

Equations (24 and 25) govern the total yaw force, Yaw_Bal.

$$
\begin{gather*}
F Y a w=[(F y R F-F y L F) \times t F \times \\
\sin (\alpha M a x F)+(F y R R-F y L R) \times t R \times \sin (\alpha M a x R)]  \tag{24}\\
Y_{\text {aw }} \text { Bal }=[A \times F y F \times \cos (\alpha M a x F)]- \\
{[B \times F y R \times \cos (\alpha M a x R)]+F Y a w} \tag{25}
\end{gather*}
$$

Equations (26 and 27) are used to apply yaw balance, Yaw_Bal = 0. If Yaw_Bal < 0, Equation (22) provides the essential tuning, while Equation (23) is used to correct the error for Yaw_Bal > 0 .

$$
\begin{align*}
& F y F=\frac{[(1-A) \times F y R \times \cos (\alpha M a x R)]-F Y a w}{A \times \cos (\alpha M a x F)}  \tag{26}\\
& F y R=\frac{[A \times F y F \times \cos (\alpha M a x F)]+F Y a w}{B \times \cos (\alpha M a x R)} \tag{27}
\end{align*}
$$

Equation (28) calculates total sideways force, Fy, as a sum of forward-facing and back sideways forces. Then, Equations (29 and 30) are used to govern the resultant speed, u , and lap time, T respectively.

$$
\begin{equation*}
F y=F y F+F y R \tag{28}
\end{equation*}
$$

$$
\begin{align*}
& u=\sqrt{\left|\frac{F y \times \text { Radius }}{m}\right|}  \tag{29}\\
& T=\frac{2 \pi \times \text { Radius }}{u} \tag{30}
\end{align*}
$$

## IV. SENSITIVITY ANALYSIS

Sensitivity means if some variables of the system are changed then how much change takes place in the system. In this project LHS is used for sampling within given design space and then the objective function is computed for different perturbations of design variables.

| LHCS | A | B | C | K | M | H | T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.556 | 0.314 | 0.372 | 0.608 | 33.916 | 1.222 | 21.001 |
|  | $0.534$ | $0.302$ | $0.357$ | 0.584 | 32.586 | 1.174 | 18.176 |
| GA | 0.301 | 0.356 | 0.497 | 0.525 | 32.174 | 1.101 | 14.7 |
| $2 \%$ | 0.307 | 0.363 | 0.507 | 0.536 | 32.818 | 1.123 | 14.846 |
| -2\% | 0.295 | 0.349 | 0.487 | 0.515 | 31.531 | 1.079 | 14.552 |
| SA | 0.3 | 0.569 | 0.444 | 0.335 | 30 | 1.255 | 15.620 |
| 2\% | 0.306 | 0.580 | 0.452 | 0.341 | 30.6 | 1.280 | 15.776 |
| -2\% | 0.294 | 0.557 | 0.435 | 0.328 | 29.4 | 1.229 | 15.463 |
| PSO | 0.3 | 0.349 | 0.487 | 0.515 | 30 | 1.08 | 13.537 |
| $2 \%$ | 0.306 | 0.355 | 0.496 | 0.525 | 30.6 | 1.101 | 13.673 |
| -2\% | 0.294 | 0.342 | 0.477 | 0.504 | 29.4 | 1.058 | 13.4 |

Pre sensitivity analysis is performed to check for properties of the perturbations in design parameters on the objective function before applying optimization
techniques. 500 samples were generated from the design space. In figure 3, 500 samples of all seven variables are shown and it shows the dependency of each variable onto the other variable. As a result we easily identify the tradeoff between variables since it clearly demonstrate the importance with respect to other variables. It also shows the variable search space for optimal solution.

Post sensitivity analysis is performed to check for special effects of the perturbations in design parameters on the objective function after getting the optimal solution. In post sensitivity analysis the system was perturbed by $2 \%$ to $-2 \%$. Some tabular and graphical results are given below.

## V. RESULTS

The conclusions obtained previously from collaborative optimization are compared with outcomes obtained from iterative technique with Latin hyper cube sampling and GA, SA and PSO to race car design problem. In the first method we generated 500 random samples from Latin hyper cube sample. To get the optimal configuration for the race car design. The Latin hyper cube sampling is used to perform the optimization. The Yaw Balance restrictions are included in the objective function of the optimization formulation.

In the second method the objective function including the Yaw balance constraint is taken as the fitness function for genetic algorithm. The size of population for genetic algorithm is 20 and generation size is 60 . The stopping criteria for genetic algorithm are up to the stalling of the result. After some time values for lap time started to stall and at that time genetic algorithm performed the iterations and generated the results of output variable. The simulated annealing and particle swarm optimization is used to compare the outcomes.

Table 5 summarizes the result for collaborative optimization formulation, iterative technique with Latin hypercube sampling and genetic algorithm. Table 6 further compare the results of different optimization techniques. After this conclusion we obtained the best technique for the similar kind of problems.

Table 4 represent the post sensitive analysis of the result obtained by different optimization techniques according to the variables value obtained through optimization.

In figure 4, function value represents lap time of car which is also representing the global best of the system verses


Fig. 4 Result of particle swam optimization
iteration. It shows search is more optimal with increasing no of iteration.

Figure 5 represent the graph of lap time on vertical axes and no of times it goes through the procedure to consolidate the solution to achieve optimal result.

Figure 6 represent the lap time on vertical axis and generation on horizontal axis, it shows that as generation passes the optimal value of lap time improves. Two type of result appeared on graph one is best value and other is mean value.

Table 5 shows that PSO obtained better result when we compare with other result or from previous paper result.


Fig. 5. Result of Simulated Annealing

TABLE 5. Comparison between Lhcs, $\mathrm{Ga}, \mathrm{Co}, \mathrm{Sa} \&$ Pso

| Parameter | Collaborative | LHCS | Genetic <br> Algorithm | Simulated <br> Annealing | P.S <br> Optimization |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| A | 0.300 | 0.556 | 0.301 | 0.300 | 0.3 |
| C | 0.574 | 0.372 | 0.497 | 0.444 | 0.487 |
| K | 0.301 | 0.608 | 0.525 | 0.335 | 0.515 |
| B | 0.3 | 0.314 | 0.356 | 0.569 | 0.349 |
| MASS | 41.7 | 33.916 | 32.174 | 30 | 30 |
| H | 1.167 | 1.222 | 1.101 | 1.255 | 1.08 |
| Radius | 400 | 384.771 | 365.329 | 400 | 400 |
| Lap | 14.884 s | 21.00 s | 14.7 s | 15.620 s | 13.537 s |
| Time |  |  |  |  |  |



Fig. 6 Genetic Algorithm Result

## VI. CONCLUSIONS

By comparing the results, we conclude that the results obtained by particle swarm optimization (PSO) are much suitable to reduce time to cover entire lap and the coefficient result generated by this technique are less effected by perturbations as compared with the results obtained from genetic algorithm (GA) and simulated annealing (SA). Race car configurations obtained from swarm optimization (PSO) are more optimal as compared to genetic algorithm (GA) and simulated annealing (SA). The comparison of our results with that of collaborative optimization shown in "table 5" shows that the results obtained in this part of research are more optimal and may increase the speed of the car as it's our primary objective.

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[^1]:    A. Abbreviations and Acronyms
    $A=$ Weight Scattering coefficient

