LS Estimation: How to make it Robust against the Increase in Channel Order

Latif Ullah Khan, Nasru Minallah, Naeem Khan

***Abstract* – In the last few decades, a tremendous increase in the mobile data traffic was observed and the number of mobile subscribers will reach to 9.038 billion in 2020. In order to cope with extensive increase in the mobile data traffic, a spectrally efficient communication technique such as Orthogonal Frequency Division Multiplexing (OFDM) is a suitable candidate to be employed at the physical layer. In OFDM, the deterioration in bit error rate (BER) performance is caused by the multipath fading channel along with channel noise. Therefore, the channel estimation technique needs to be employed to minimize the degradation caused by the fading channel. A Least Square (LS) estimation is a type of estimation that does not utilize the statistics of the channel and can be employed to cancel out the degradation effects of the wireless channel. The deterioration in performance due to fading channel is proportional to the channel order. The performance of the LS estimation is susceptible to degradation proportional to the rise in order of the channel. This paper aims at making the performance of the LS estimator robust against the increase in channel order. Simulation results confirmed our modifications regarding the robustness of the LS estimation in OFDM system.**

***Index Terms* – OFDM, LS Estimation, comb-type channel estimation, MSE.**

I. INTRODUCTION

Due to a gigantic increase in the number of mobile subscribers accessing the cellular networks and popularity of social media sites such as Facebook and Twitter, a tremendous increase in mobile data traffic was observed in the last few decades. In order to tackle the capacity issues, it is imperative to utilize bandwidth efficient communication schemes. OFDM is a preferable candidate to be used at the physical layer to improve the system capacity. In frequency domain, the efficiency in terms of bandwidth is achieved using the overlapping characteristic of the orthogonal sub carriers. Widespread application of OFDM includes Wi-Max, Wi-Fi, power line communications and Long Term Evolution (LTE) [1][2][9]. The significant advantage of the multicarrier communication techniques (such as OFDM) is the usage of the equalization techniques with low computational complexity due to the frequency domain operations nature while complex equalization techniques were employed in single carrier communication schemes.

In wireless communication, the signal suffers from reflections, diffraction and scattering and thus, multipath fading results which significantly degrade the performance of the signal. Estimating the channel impulse response and then performing equalization on the received signal is a possible solution for minimization of the adverse effects of the multipath fading channel. Estima tion is considered to be an imperative part of the wireless communication system. Without estimation, the received signal quality will be highly degraded in case of OFDM system. Estimation of the channel impulse response can be done by utilization of the Minimum Mean Square Error (MMSE) estimator and LS estimator respectively at pilot locations. The insertion fashion of pilots in OFDM can be broadly divided into two types such as Comb-type Channel Estimation (CCE) and Block-type Channel Estimation (BCE) depending on the variations rate of the multipath fading channel. A CCE is adopted for the fast fading channel while a BCE is used for the fading channel with slow variations in its impulse response. The channel estimation in OFDM can divided into blind, semi-blind or pilot aided depending on the density of the pilot symbols per OFDM symbol. The pilots can be used with blind channel estimation to improve its performance at the cost of the pilot overhead. A signal-to-noise ratio gain up to 3-dB can be achieved in the OFDM using coherent PSK with accurate channel estimation over the differential PSK [3].

In literature, a lot of work has been carried out regarding the channel estimation in OFDM system [4]-[6],[10][11]. In [4], LS estimation was evaluated in terms of BER for CCE. Different one dimensional (1-D) interpolation techniques such as (low pass, linear, spline, second order and time domain) were considered. Low pass interpolation was found better among the other 1-D techniques of interpolation. Performance comparison of LS and MMSE estimator revealed the enhanced performance of the MMSE estimator; however, this will be at the cost of the usage of channel statistics [5]. In [10], a new algorithm was proposed that was based on modifications of the LS estimation technique. In [6], a robust estimation algorithm was proposed using alterations in the MMSE estimation. The alterations in the MMSE estimator were based on making changes in the observation matrix. The modified MMSE estimator was tested for different channels and stability of the proposed scheme was observed through the simulation results.

In this paper, alterations in the LS Estimation have been made in order to make its performance, robust against the increase in channel order. Comparing the BER Vs SNR curves for the LS estimator and M-LS estimator revealed the improved performance of the proposed channel estimation technique.

II. SYSTEM MODEL

Consider Fig. 1 illustrating an OFDM system model with channel estimation. Firstly, we have to transform the bits into a frequency domain sequence of symbols using

LatifUllah Khan and Naeem Khan are with Department of Electrical Engineering and Nasru Minallah is with Department of Computer System Engineering, University of Engineering and Technology, UET Peshawar, Pakistan.
Email: latifullahkhan@uetpeshawar.edu.pk, naeem.engr2000@gmail.com, minallah.nasru@gmail.com.
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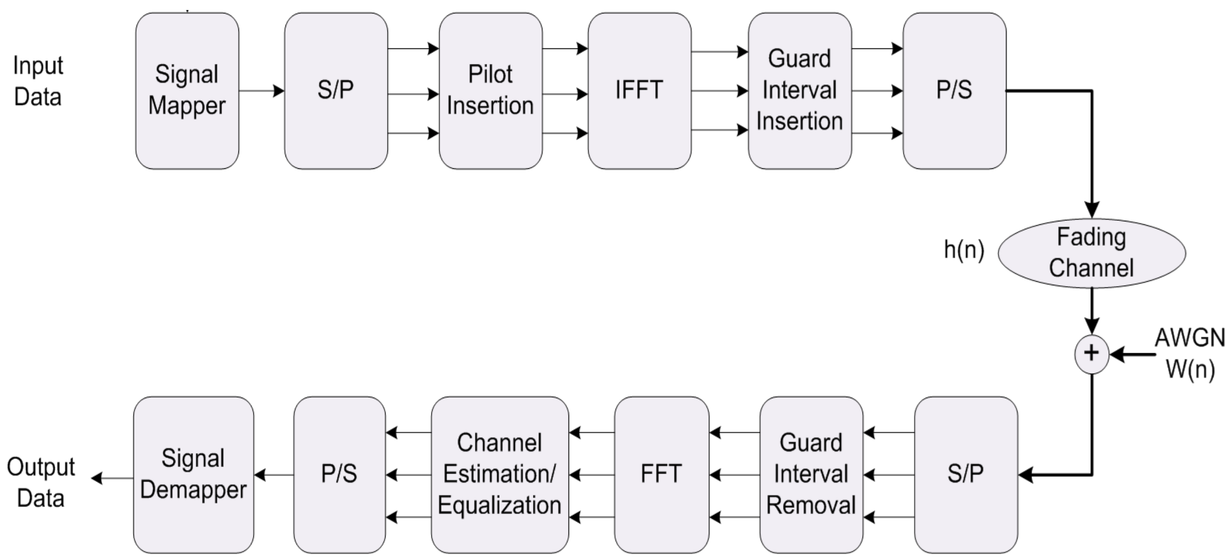

Fig. 1 System Model

mapper block. Then, the conversion of the serial stream of mapped symbols into parallel is performed.

$$Z(k) = [Z(0), Z(1), Z(2), \dots, Z(N_D - 1)]^T \quad (1)$$

Where N_D is the number of data symbols in a particular OFDM symbol. After mapping, the conversion from serial to parallel is done and then N_P pilot tones are positioned into OFDM symbols using comb-type pilot fashion. Let the data subcarriers indices set is given by $I_s = \{n_i\}$ $(where\ i = 0,1,\dots,N_D - 1)$and set of pilot indices is given by $I_p = \{n_j\}$ $(where\ j = 0,1,\dots,N_P - 1)$. In a given OFDM symbol $Z(n)$, the pilot symbols $Z_p(n)$ are given by:

$$Z_p(n) = Z(n) \qquad n \in I_p \quad (2)$$

Similarly, in a given OFDM symbol $Z(n)$, the symbols carrying data $Z_s(n)$ are given by:

$$Z_s(n) = Z(n) \qquad n \in I_s \quad (3)$$

The output of the IFFT block is given by

$$z(n) = \sum_{k=0}^{N-1} Z(k)\, e^{j2\Pi nk/N} \; n = 0,1,2,\dots,N-1 \quad (4)$$

The cyclic prefix addition is performed on the signal and then, passed through the channel. On the receiver side, the signal is passed through the FFT block after performing the removal of cyclic prefix to yield the frequency domain signal. The output of the FFT block is given by:

$$R(k) = \sum_{n=0}^{N-1} r(n) e^{-\frac{j2\Pi kn}{N}} \quad k = 0,1,2,\dots,N-1 \quad (5)$$

Then the channel estimation and equalization is performed on the received signal to improve its quality. Finally, the signal is fed to demapper to yield the output bits.

III. CHANNEL ESTIMATION & EQUALIZATION ALGORITHM

Channel estimation is considered to be an imperative part of the wireless communication system. The signal in the wireless channel suffers from diffraction, reflection and scattering, which results in multipath fading, which degrade the performance of the communication system. Therefore, it is imperative to cancel out the degradation caused by the multipath fading to improve the quality of service (QoS). Depending on the pilot tones density in a given OFDM symbol, the estimation can be categorized into three categories such as (a) blind, (b) pilot assisted and (c) semi blind channel estimation. The number of pilots in pilot assisted is higher than semi-blind channel estimation with superior performance. Blind channel estimation utilizes the received signal statistical information to minimize the negative impairments of the fading channel. Fig. 2 depicts the types of channel estimation schemes in OFDM.

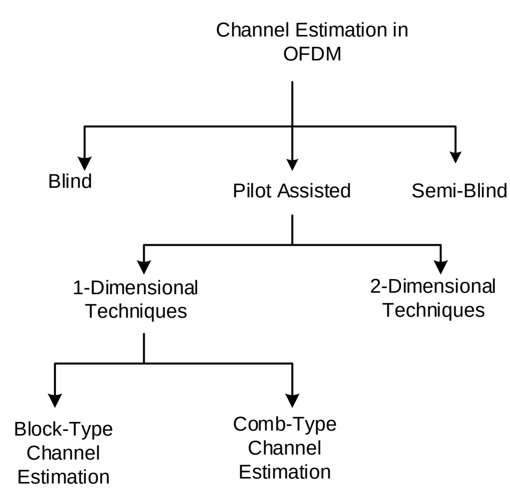

Fig. 2 Types of Channel Estimation

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The insertion of pilot tones in the CCE is based on the following equation:

$$Z(K) = Z(nL+q) = \begin{cases} Z_P(n) & q = 0 \\ inf.data & q = 1,2,3,\ldots L-1 \end{cases} \quad (6)$$

Where

$Z_P(n)$ is the n[th] pilot carrier value.

$L = \frac{total\ number\ subcarriers}{total\ Pilots}$

The pilot spacing in CCE is depicted in Fig. 3.

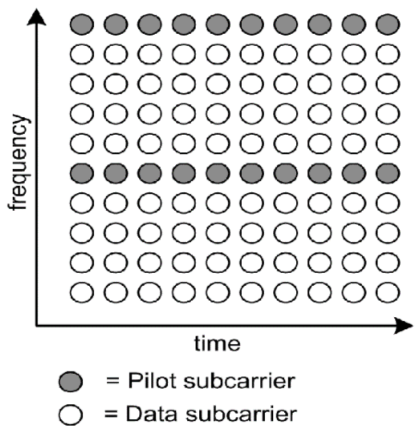

Fig. 3 Comb-type Channel Estimation

A. Conventional LS estimator

The LS estimate is given by the following equation [7]:

$$H_{LS} = \frac{R_P(k)}{Z_P(k)} \quad k = 0,1,\ldots,N_P-1 \quad (7)$$

Where $R_P(k)$ and $Z_P(k)$ are the k[th] received and k[th] transmitted pilot respectively.

Or

$$H_{LS} = (diag(Z_P))^{-1}R_P \quad (8)$$

Where R_P and Z_P are the received and transmitted pilot symbol vectors respectively.

Or

$$H_{LS} = R_P \odot ((Z_P) \bullet^{\wedge -1}) \quad (9)$$

Where $\odot$ and $(\ ) \bullet^{\wedge -1}$ represent the element wise multiplication and element wise exponentiation operations respectively.

Or

$$H_{LS} = [R_P(0) * (Z_P(0))^{-1}\ ,\ R_P(1) * (Z_P(1))^{-1}, \ldots, R_P(N_P - 1) * \ (Z_p(N_P-1))^{-1}]^T \quad (10)$$

There is spacing between the pilots in case of CCE therefore; some interpolation techniques should be used. This paper utilizes one-dimensional (1-D) interpolation techniques to compute missing impulse response at data tones. The reason for the utilization of the 1-D interpolation is due to fact of lower computational complexity than 2-D interpolation schemes.

B. *Modified LS Estimator*

The LS estimator is based on minimizing the square of the difference between the detected and estimated signal. The LS estimation does not consider the effect of the channel noise while estimating the impulse response of the channel and thus, susceptible to performance degradation due to channel noise. The received signal can be given by:

$$r = ZH + W \quad (11)$$

Where

$Z = diag\{Z(0), Z(1), \ldots, Z(N-1)\}$

$H = [H(0), H(1), \ldots, H(N-1)]^T$ is the channel frequency response vector.

$W = [W(0), W(1), \ldots, W(N-1)]^T$ is the AWGN vector.

Let $F_L[k,n] = \frac{1}{\sqrt{N}} e^{\frac{j2\pi(k)(n)}{N}}$ for $n = 0,1,2,3,\ldots,N-1$ and $k = 0,1,2,3,..L-1$ and the columns of F_L proportional to pilot positions are combined into a matrix F_Q respectively.

For pilot symbols (11) becomes:

$$r_P = Z_p F_Q{}^H h_P + W_P \quad (12)$$

The expression for the estimated channel frequency response by M-LS estimator can be found by minimizing the following expression:

$$e(h_P) = (r_P - Z_p F_Q{}^H h_P)^H (r_P - Z_p F_Q{}^H h_P) \quad (13)$$

$$e(h_P) = r_P{}^H r_P - r_P{}^H Z_p F_Q{}^H h_P - h_P{}^H F_Q Z_P{}^H r_P + h_P{}^H F_Q Z_P{}^H Z_p F_Q{}^H h_P \quad (14)$$

$r_P{}^H Z_p F_Q{}^H h_P$ and $h_P{}^H F_Q Z_P{}^H r_P$ are both equal, therefore (14) becomes:

$$e(h_P) = r_P{}^H r_P - 2h_P{}^H F_Q Z_P{}^H r_P + h_P{}^H F_Q Z_P{}^H Z_p F_Q{}^H h_P \quad (15)$$

The gradient is

$$\frac{\partial e(h_P)}{\partial h_P} = -2F_Q Z_P{}^H r_P + 2F_Q Z_P{}^H Z_p F_Q{}^H h_P \quad (16)$$

Setting the $\frac{\partial e(h_P)}{\partial h_P} = 0$, (16) becomes

$$h_P = \left(F_Q Z_P{}^H Z_p F_Q{}^H\right)^{-1} (F_Q Z_p{}^H) r_P \quad (17)$$

$$H = F_L{}^H h_P \quad (18)$$

(18) performs the transformation from time into the frequency domain and also performs interpolation and will lead to improved M-LS estimation at all frequencies. In case of LS estimation with interpolation techniques, the increase

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in channel order causes the increase in distortion which further degrades the estimation performance because of the corresponding increase in interpolation error.

IV. SIMULATION RESULTS

In this section, the results of the simulations carried out in MATLAB ® are presented. The number of sub carriers utilized in simulation is 512. The modulation schemes considered are BPSK, QPSK, 32-QAM, 16-QAM and 64-QAM respectively. The fast fading Rayleigh channel is employed in the presence of AWGN for evaluating the OFDM system performance. Fig. 4 to 7 shows the performance of the estimation schemes in OFDM (such as proposed and LS estimation with interpolation techniques) for different channels orders. The channel used in this paper consists of L uncorrelated complex Gaussian random taps with zero mean. The variance of each tap in case of exponential power delay profile (EPDP) is given by:

$$\sigma_l^2 = e^{-\frac{l}{45}} \qquad l \epsilon \, [0, L) \tag{19}$$

The results of Fig. 4 to 6 illustrates that the proposed estimation algorithm outperformed the LS estimation with different interpolation techniques.

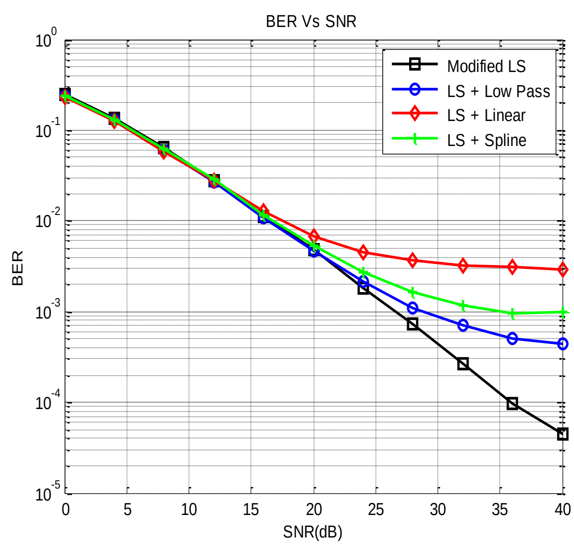

Fig. 4 Performance of OFDM system using BPSK for channel order 20

LS estimator incurs degradation in performance with rise in order of the fading channel for CCE [8] as illustrated in Fig. 6 however, M-LS estimator has stable performance against the rise in order of the channel and the system becomes useless for higher channel order in case of the LS estimation. Fig. 7 illustrates the OFDM system performance using various digital modulation schemes. Fig. 7 depicts the degradation in BER for higher order digital modulation techniques. The reason for such loss in BER performance is due to placement of the constellation points in a close proximity to each other.

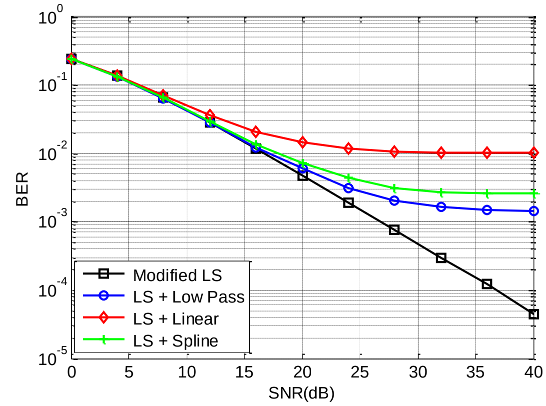

Fig. 5 Performance of OFDM system using BPSK for channel order 26

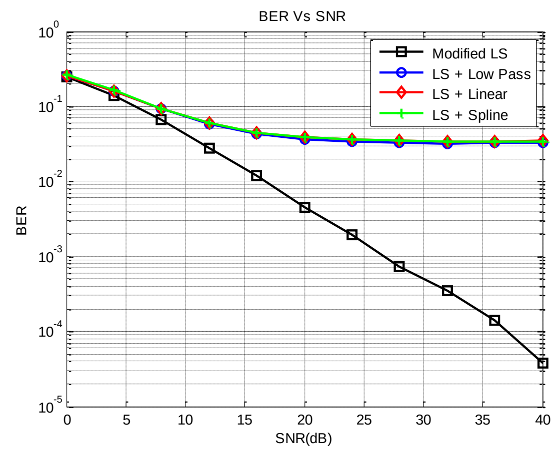

Fig. 6 Performance of OFDM system using BPSK for channel order 40

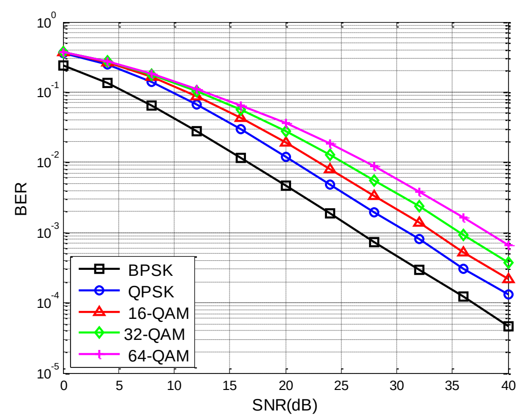

Fig. 7 Comparison curves of OFDM system using M-LS for different modulation schemes for channel order 20

V. CONCLUSION

In this paper, modifications in the LS estimator are proposed and the performance tests under different channel conditions are provided. The rise in order of the channel has proportional distortion effect on the signal. The M-LS estimator is observed to have robust performance against the rise in order of the channel while the LS estimator using 1-D

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interpolation for comb-type channel estimation has degraded performance for higher channel orders.

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