

# Robust Control of Slow Time Varying System utilizing Predictive Model

Ali Akbar Siddique, M. Yasir Zaheen and M. Tahir Qadri

**Abstract** – Recent years attracted the research interest in the adaptation of the system with respect to the given reference input in real time. It is a well-known fact that every system available today is time varying. In order to control these system, it is important to introduce a control algorithm suitable to tackle such system in real time applications. For this purpose a method to control such systems using Model predictive controller (MPC) is proposed in this paper to control the slow time varying first order system with ( $\tau$ ) as a time dependent variable. MPC is capable of predicting the changes by utilizing the previous reading and alter the parameter to track that change few steps ahead. System variations are not limited to its order, but the proposed model possess a time varying coefficient. Proposed model is compared with the existing PID controller and tracked the disturbance faster than PID controller.

**Index Term** – Model Predictive Controller (MPC), Slow Time varying System, PID Controller, Predictive Model, State Space Model.

## I. INTRODUCTION

In recent times, field of control engineering is progressing at an exponential rate. Investment and research in this field is also increasing on the daily bases due its application in industry. With the recent advancements in complex time varying system used in industry, there is an urgent need for the control mechanism capable of controlling them in real time.

There are various algorithms used that are common for predicting the system response and to control them. Finite Impulse Response (FIR) filter is the most favorable among them, it is appealing to many control engineers due to its proper representation of response time and gain. On the downside, it require large model order d around 30 (thirty) to 60 (sixty) coefficients depending on the system dynamics [1-6]. Stochastic gradient method is also utilized by many researchers to design a control algorithm for time variant systems [7-9]. Over the years many control algorithm are introduced capable of predicting the trajectory using the output and the current state of the system such as Least Mean Square (LMS) Algorithm, Fuzzy abased algorithm etc [10-14].

Neural network (NN) is also used for controlling slow system that are time dependent [15-19] and includes certain delay parameters. In the proposed model, a model based on predictive algorithm is utilized to control the slow time varying first order system. NN require weighted coefficients to track the trajectory of the variable [20-23].

Ali Akbar Siddique is with Department of Telecommunication Engineering, M. Yasir Zaheen and M. Tahir Qadri are with Department of Electronics Engineering, Sir Syed University of Engineering & Technology, Karachi, Pakistan. Email: ali124k@hotmail.com, myasir@ssuet.edu.pk, mtahirq@hotmail.com. Manuscript received on July 20, 2018, revised on July 24, 2019 and accepted on Aug 23, 2019.

## II. MATHEMATICAL MODEL

The model of a first order time varying system is represented in equation 10, where  $\tau$  is a time varying parameters that tends to change over time [26-27]. These type of system achieve their steady state after a very long time and it is imperative to make them fast so they may perform their task more effectively.

$$G(s, \tau) = \frac{1}{\tau s + 1} \quad (1a)$$

$$g(t, \tau) = \frac{1}{\tau} e^{-\frac{1}{\tau}t} \quad (1b)$$

Usually temperature based system are slow system and it varies very slowly in controlled environment but if an unexpected incident occurs like gas leakage temperature of that particular surrounding may change at an accelerated rate.

## III. PREDICTIVE CONTROLLER MODEL

In the proposed work, a time varying system is represented in terms of state space model expressed in equations 2 & 3.

$$x(k+1) = A_q x_q(k) + B_q u(k) \quad (2)$$

$$y(k) = C_q x_q(k) + D_q u(k) \quad (3)$$

For the purpose of receding horizon control, it is imperative to know the current information of the desired plant model for prediction and control. Parameter  $u(k)$  is basically an input of the system and  $y(k)$  is a process or plant output as elaborated in equations 2 & 3. Implicitly assuming the system output is unaffected by input at a given time instant [28-30].

The next step is to devise a mathematical model of a predictive control system capable of calculating the predicted output by utilizing an upcoming control signal as manipulated variables. Control model has an optimization window known as prediction horizon  $N_P$ , considering  $k_i$  be the current time instant. Prediction horizon is basically a total length of an optimization window and  $N_C$  is termed as a control horizon identifying the samples which will used to capture the future control trajectory. All the predicted parameters accumulated are in terms of state variable matrix  $x(k_i)$  and the future control trajectory  $u(k_i + j)$ , where  $j = 0, 1, 2, \dots, N_C - 1$ .

$$U = [u(k_i) \ u(k_i + 1) \ \dots \ u(k_i + N_c - 1)]^T \quad (4)$$

$$Y = [y(k_i + 1) \ y(k_i + 2) \ \dots \ y(k_i + N_p)]^T \quad (5)$$

Where  $U$  is a control trajectory depended on control horizon expressed in equation 7 and  $Y$  are the output coefficients with the dimension equal to prediction horizon and equation 6 displays the expression of the process output.

$$Y = Fx(k_i) + \Phi U \quad (6)$$

$$U = (\Phi^T \Phi + \bar{R})^{-1} (\Phi^T R_S - \Phi^T Fx(k_i)) \quad (7)$$

Where,

$$F = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix} \quad (8)$$

$$\Phi = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & CA^{N_p-N_c}B \end{bmatrix} \quad (9)$$

$R_S$  is basically a vector containing the information about the set-points or reference point with the same length as predictive horizon.  $\bar{R}$  is a tuning parameter in a form of a diagonal matrix and has the same length as prediction horizon containing the constant value of 1.

#### IV. OPTIMIZATION TECHNIQUE

For the given reference parameters at a time instant  $k_i$ , objective of the model predictive controller is to converge the predicted output as close as possible to the desired input or set parameter. At this point the control vector begins to find the best possible control signal  $U$  to get the minimum error between both set-point & the predicted output [31-32].

$$J = (R_S - Y)^T (R_S - Y) + U^T \bar{R} U \quad (10)$$

In equation 10,  $J$  represents the cost function that needs to be minimize for better output response close to the set-point.  $(R_S - Y)^T (R_S - Y)$  in equation 9 is used to minimize the error by subtracting the predicted output and set-point signal and the other term  $U^T \bar{R} U$  controls the error signal by trying to find the values that makes the cost function zero or close to zero.

#### V. RESULTS

After the implementation of the controller with time varying model, immediately it acquires an output response with some error and as the time passes, controller utilize the reference signal and recently

acquired output to reduce the error until it becomes negligible or until the system changes completely.

For the Predictive controller, value of  $(N_p)$  is assumed to be 8, it is basically an optimization window for the Predictive horizon and 4 was selected for the control horizon ( $N_c$ ). Control Signal was discussed in section 2 equation 6 having  $\Phi^T \Phi$ ,  $\Phi^T F$  and  $\Phi^T \bar{R}$  matrices, for precise calculation of the control signal these matrices plays a vital role in reducing the cost function ( $J$ ). Equations-11, 12 and 13 are the actual values accumulated using the first order model, they are utilized to calculate the control signal ( $U$ ). We can observe that the values of control signal is close to zero, it's because the output ( $Y$ ) has already converged with the set-point or reference point ( $\bar{R}_S$ ), fig-1 unveil the convergence of the control and output signal.

$$\Phi^T \Phi = \begin{bmatrix} 165.54 & 134.52 & 104.67 & 76.86 \\ 134.52 & 110.5 & 86.91 & 64.46 \\ 104.67 & 86.01 & 69.31 & 54.12 \\ 76.86 & 64.46 & 52.12 & 39.94 \end{bmatrix} \quad (11)$$

$$\Phi^T F = \begin{bmatrix} 963.38 & 183.58 & 31.683 \\ 782.89 & 148.51 & 24.265 \\ 609.13 & 114.99 & 17.846 \\ 447.29 & 84.071 & 12.427 \end{bmatrix} \quad (12)$$

$$\Phi^T \bar{R}_S = \begin{bmatrix} 31.683 \\ 24.265 \\ 17.846 \\ 12.427 \end{bmatrix} \quad (13)$$

$$U = \begin{bmatrix} -0.903e^{-09} \\ 0.0027e^{-09} \\ 0.1500e^{-09} \\ 0.0322e^{-09} \end{bmatrix} \quad (14)$$

Values of the control signal in equation 14 are extremely low due to the fact that the system output has already attain the approximate reference point with minimum error and will not require further extensive computation. Fig-1 shows the step responses of proposed time varying model for all the values of  $\tau$ , fig-2 is the representation of control & output signal for whole time period and the changes occurs in its time frame.

Basically fig-2 displays the control and output signals at each value of  $\tau$  with respect to their control and output signal. Different values of  $\tau$  represent the changes occurring after an instant of time which may cause a certain change in system behavior. The value of control signal is in initially high because it is adjusting itself as it reduces the error to zero.

By observing the samples of fig-2, controller require little effort to predict and track the reference trajectory that is because the value of  $\tau$  is 1 which makes the system pole further away from the origin making it easy to track. On the other hand, if the value of  $\tau$  changes to 100, the system pole at this point is close to the origin, although the system is stable but the predictive controller will require much more effort to track the reference point and to make the response faster. For further analysis of the proposed control algorithm, a small disturbance in the system was introduced. The reason

for applying this disturbance is to observe how fast the controller track and control it. It can be observed in fig-3, the disturbance in the system output occurs at a time instant of about 27 and the controller adapts according to its effect very quickly. In fig-4, a Proportional Integrator and Differentiator is introduced as a baseline controller to control the response of the slow time varying system, it was used to compare the response time of both controllers. PID takes more time to track the initial response, it approximately takes 25 second when  $\tau$  is set to 10 as shown in fig-4 and on the other hand it has to make a lot of effort to make it stable and free of any steady state error, while MPC takes less effort to perform the same task as given in fig-2 as a blue line for  $\tau=10$ .

## VI. CONCLUSION

Proposed control algorithm of (MPC) is implemented using MATLAB utilizing a time varying model. Time varying system is assumed to change with the change of  $\tau$  having the values of 1, 10 and 100 but the controller implemented tracked the reference point by utilizing the output signal and reducing error to the point it will become negligible until the next change in  $\tau$  occurs. The designed control algorithm performed according to the expectation and will be useful for application in which the base system model tends to change frequently.

## REFERENCES

- [1] Hsieh, Hsin-Ju, et al. "Linear prediction filtering on cepstral time series for noise-robust speech recognition." *Consumer Electronics-Taiwan (ICCE-TW)*, 2016 IEEE International Conference on. IEEE, 2016.
- [2] Davidson, Jonathan N., David A. Stone, and Martin P. Foster. "Real-Time Prediction of Power Electronic Device Temperatures Using PRBS-Generated Frequency-Domain Thermal Cross Coupling Characteristics." *IEEE Transactions on Power Electronics* 30.6 (2015): 2950-2961.
- [3] Szadkowski, Zbigniew, and D. Glas. "Adaptive linear predictor FIR filter based on the Cyclone V FPGA with HPS to reduce narrow band RFI in AERA radio detection of cosmic rays." 2015 4th International Conference on Advancements in Nuclear Instrumentation Measurement Methods and their Applications (ANIMMA). IEEE, 2015.
- [4] Pak, Jung Min, et al. "Self-recovering extended Kalman filtering algorithm based on model-based diagnosis and resetting using an assisting FIR filter." *Neurocomputing* 173 (2016): 645-658.
- [5] Pak, Jung Min, et al. "Switching extensible FIR filter bank for adaptive horizon state estimation with application." *IEEE Transactions on Control Systems Technology* 24.3 (2016): 1052-1058.
- [6] Zhao, Shunyi, et al. "Minimum variance unbiased FIR filter for discrete time-variant systems." *Automatica* 53 (2015): 355-361.
- [7] Ma, Yudong, Jadranko Matuško, and Francesco Borrelli. "Stochastic model predictive control for building HVAC systems: Complexity and conservatism." *IEEE Transactions on Control Systems Technology* 23.1 (2015): 101-116.
- [8] Qu, Ting, et al. "Switching-based stochastic model predictive control approach for modeling driver steering skill." *IEEE Transactions on Intelligent Transportation Systems* 16.1 (2015): 365-375.
- [9] Ding, Feng, Ling Xu, and Quanmin Zhu. "Performance analysis of the generalised projection identification for time-varying systems." *IET Control Theory & Applications* 10.18 (2016): 2506-2514.
- [10] Yunlu, Li, et al. "A novel predictive control for active power filter using a variable-step LMS algorithm." *The 27th Chinese Control and Decision Conference (2015 CCDC)*. IEEE, 2015.
- [11] Bautista-Quintero, R., et al. "Close-Loop Control Identification of an Inverted Pendulum Based on Parameter Linear Regressor and Generalised Predictive Control+ Integral Compensator." *University of New Brunswick Fredericton, The 14th IFToMM World Congress, Taipei, Taiwan, NB Canada, October 25. Vol. 30. 2015.*
- [12] Ding, Derui, et al. "Finite-Horizon Control for Discrete Time-Varying Systems with Randomly Occurring Nonlinearities and Fading Measurements." *IEEE Transactions on Automatic Control* 60.9 (2015): 2488-2493.
- [13] Dong, Hongli, et al. "Event-based filter design for a class of nonlinear time-varying systems with fading channels and multiplicative noises." *IEEE Transactions on Signal Processing* 63.13 (2015): 3387-3395.
- [14] Bououden, S., Mohammed Chadli, and Hamid Reza Karimi. "An ant colony optimization-based fuzzy predictive control approach for nonlinear processes." *Information Sciences* 299 (2015): 143-158.
- [15] Afram, Abdul, et al. "Artificial neural network (ANN) based model predictive control (MPC) and optimization of HVAC systems: A state of the art review and case study of a residential HVAC system." *Energy and Buildings* 141 (2017): 96-113.
- [16] Vazquez, Sergio, et al. "Model predictive control for power converters and drives: Advances and trends." *IEEE Transactions on Industrial Electronics* 64.2 (2016): 935-947.
- [17] Rossiter, J. Anthony. *Model-based predictive control: a practical approach*. CRC press, 2017.
- [18] Delgoda, Dilini, et al. "Irrigation control based on model predictive control (MPC): Formulation of theory and validation using weather forecast data and AQUACROP model." *Environmental Modelling & Software* 78 (2016): 40-53.
- [19] Mesbah, Ali. "Stochastic model predictive control: An overview and perspectives for future research." *IEEE Control Systems Magazine* 36.6 (2016): 30-44.
- [20] Kouro, Samir, et al. "Model predictive control: MPC's role in the evolution of power electronics." *IEEE Industrial Electronics Magazine* 9.4 (2015): 8-21.
- [21] Kouvaritakis, Basil, and Mark Cannon. *Model predictive control*. Switzerland: Springer International Publishing (2016).
- [22] Serale, Gianluca, et al. "Model predictive control (MPC) for enhancing building and HVAC system energy efficiency: Problem formulation, applications and opportunities." *Energies* 11.3 (2018): 631.
- [23] Baidya, Roky, et al. "Multistep model predictive control for cascaded h-bridge inverters: Formulation and analysis." *IEEE Transactions on Power Electronics* 33.1 (2017): 876-886.
- [24] Zheng, Yang, et al. "Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies." *IEEE Transactions on Control Systems Technology* 25.3 (2016): 899-910.
- [25] Weiss, Avishai, et al. "Model predictive control for spacecraft rendezvous and docking: Strategies for handling constraints and case studies." *IEEE Transactions on Control Systems Technology* 23.4 (2015): 1638-1647.

- [26] Stellato, Bartolomeo, Tobias Geyer, and Paul J. Goulart. "High-speed finite control set model predictive control for power electronics." *IEEE Transactions on power electronics* 32.5 (2016): 4007-4020.
- [27] Morstyn, Thomas, et al. "Model predictive control for distributed microgrid battery energy storage systems." *IEEE Transactions on Control Systems Technology* 26.3 (2017): 1107-1114.
- [28] Naveau, Maximilien, et al. "A reactive walking pattern generator based on nonlinear model predictive control." *IEEE Robotics and Automation Letters* 2.1 (2016): 10-17.
- [29] Li, Xiaodi, and Jinde Cao. "An impulsive delay inequality involving unbounded time-varying delay and applications." *IEEE Transactions on Automatic Control* 62.7 (2017): 3618-3625.
- [30] Li, Yang, et al. "Time-varying system identification using an ultra-orthogonal forward regression and multiwavelet basis functions with applications to EEG." *IEEE transactions on neural networks and learning systems* 29.7 (2018): 2960-2972.
- [31] Goyal, Vishal, Vinay Kumar Deolia, and Tripti Nath Sharma. "Neural network based sliding mode control for uncertain discrete-time nonlinear systems with time-varying delay." *International Journal of Computational Intelligence Research* 12.2 (2016): 125-138.
- [32] Gao, H., et al. "Delay-dependent output-feedback stabilisation of discrete-time systems with time-varying state delay." *IEE Proceedings-Control Theory and Applications* 151.6 (2004): 691-698.
- [33] Zhang, Hong, et al. "A simple first-order shear deformation theory for vibro-acoustic analysis of the laminated rectangular fluid-structure coupling system." *Composite Structures* 201 (2018): 647-663.
- [34] Grüne, Lars, and Jürgen Pannek. "Nonlinear model predictive control." *Nonlinear Model Predictive Control*. Springer, Cham, 2017. 45-69.
- [35] Felipe, Carmen M., José L. Roldán, and Antonio L. Leal-Rodríguez. "An explanatory and predictive model for organizational agility." *Journal of Business Research* 69.10 (2016): 4624-4631.
- [36] Türker, Türker, Umit Buyukkeles, and A. Faruk Bakan. "A robust predictive current controller for PMSM drives." *IEEE Transactions on Industrial Electronics* 63.6 (2016): 3906-3914.
- [37] Gulbudak, Ozan, and Enrico Santi. "FPGA-based model predictive controller for direct matrix converter." *IEEE Transactions on Industrial Electronics* 63.7 (2016): 4560-4570.

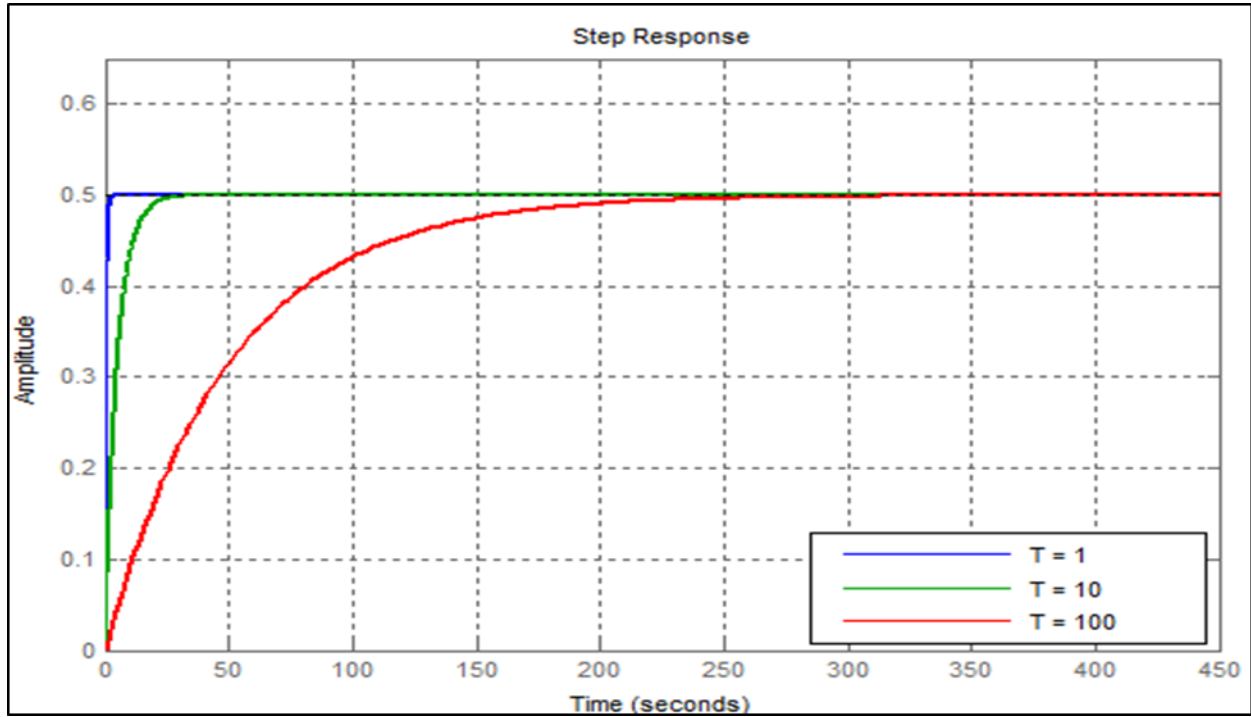


Fig.1 Step Responses of Time Varying Model for  $\tau = 1, 10, 100$

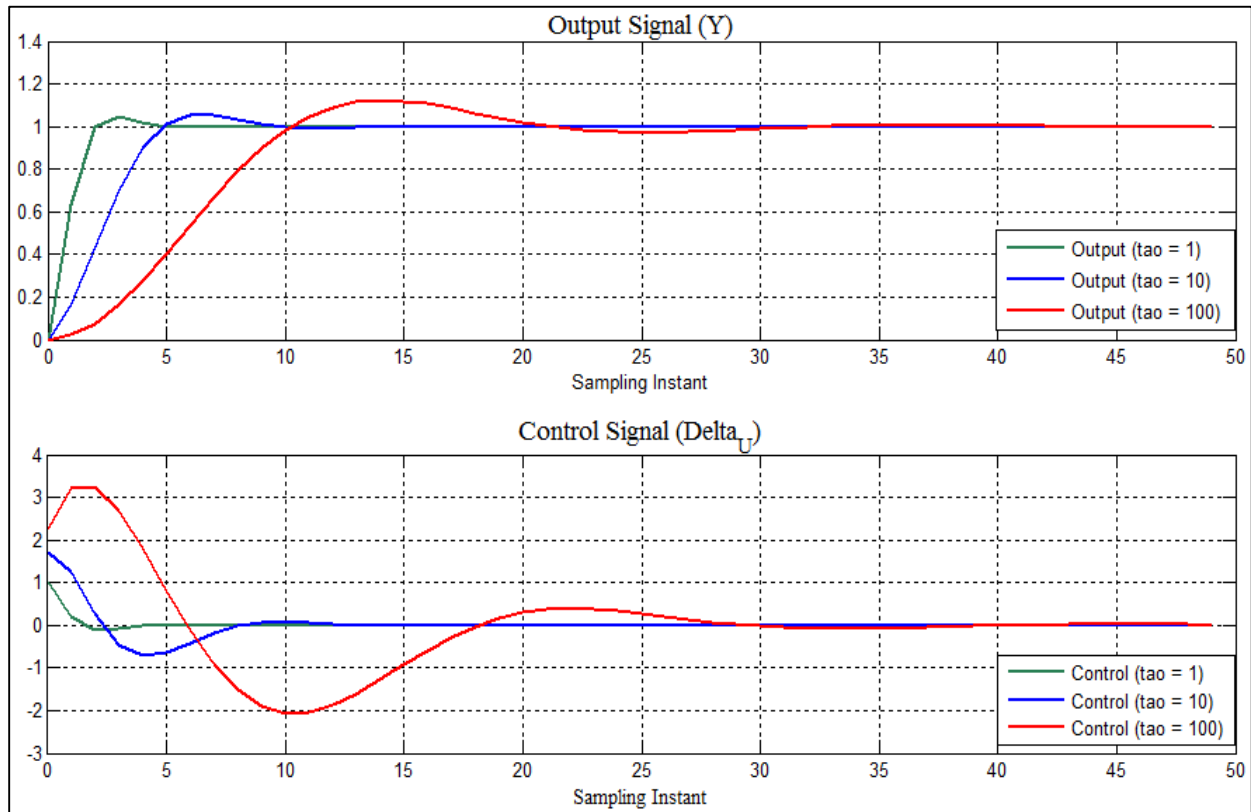


Fig.2 Control & Output Signal at  $\tau = 1, 10, 100$ .

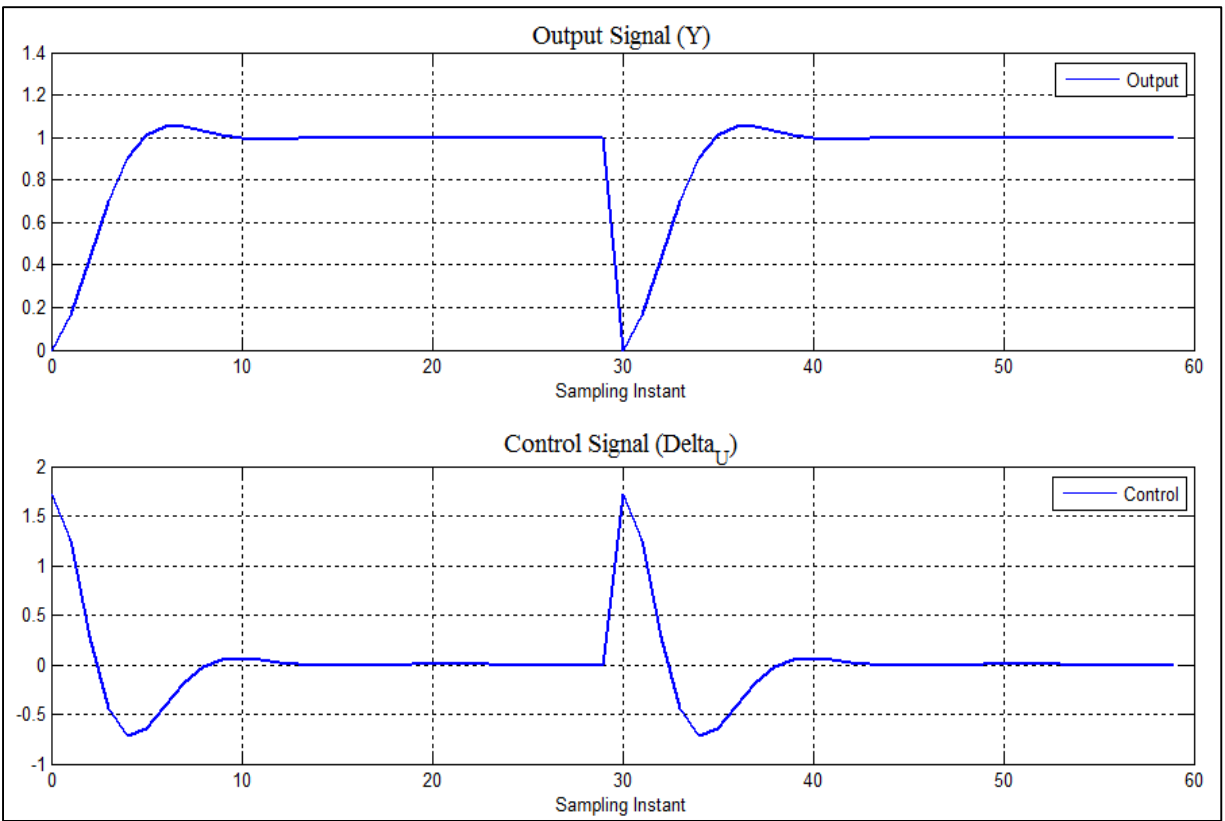


Fig.3 Control & Output Signal with disturbance

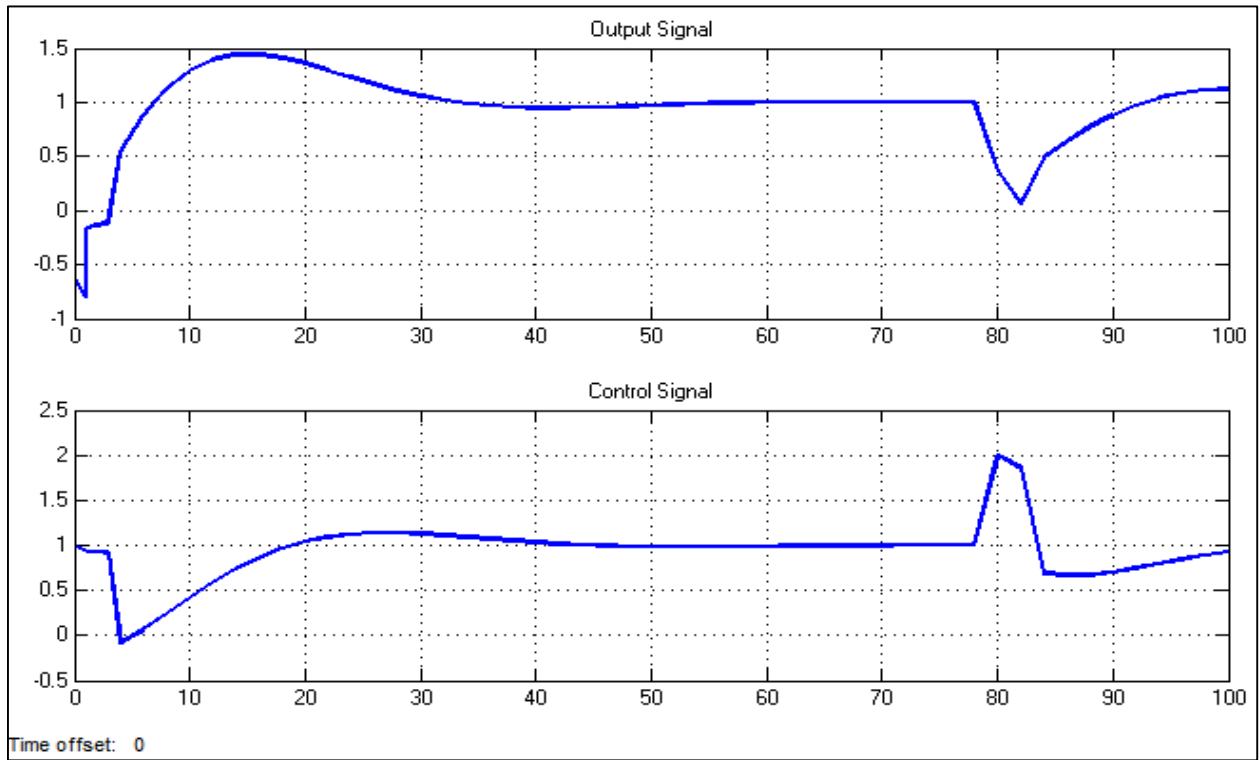


Fig. 4 Output & Control of System using PID Controller